INDUSTRIAL MATHEMATICS PRACTICALLY APPLIED

AN INSTRUCTION AND REFERENCE BOOK FOR STUDENTS IN MANUAL TRAINING, INDUSTRIAL AND TECHNICAL SCHOOLS, AND FOR HOME STUDY

BY

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WITH 250 ILLUSTRATIONS AND OVER 1000 PROBLEMS AND ANSWERS

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PREFACE

This book is the result of 12 years experience as supervisor of apprentices, designing, shop foreman and instructor in evening classes in technical schools.

The author has made every effort to simplify and analyze, step by step, the various phases of industrial mathematics. The problems, examples and illustrations have been carefully selected and chosen to make them as practical and interesting as possible so that they will stimulate and encourage the student to think clearly, reason and analyze for himself.

The first few pages are intended to be used for review or reference work at the discretion of the student, or for those who have only an elementary knowledge of mathematics.

Standard formulas and data, such as are usually found in mechanics' hand books, trade journals and technical literature, are frequently used and sufficient material in the way of explanations, illustrations and examples are given along with each subject so that the student will become familiar with them. It is hoped that such information will make this book of permanent value for reference and as a hand book.

PAUL V. FARNSWORTH.

DETROIT, MICH.
May 5, 1921.

CONTENTS

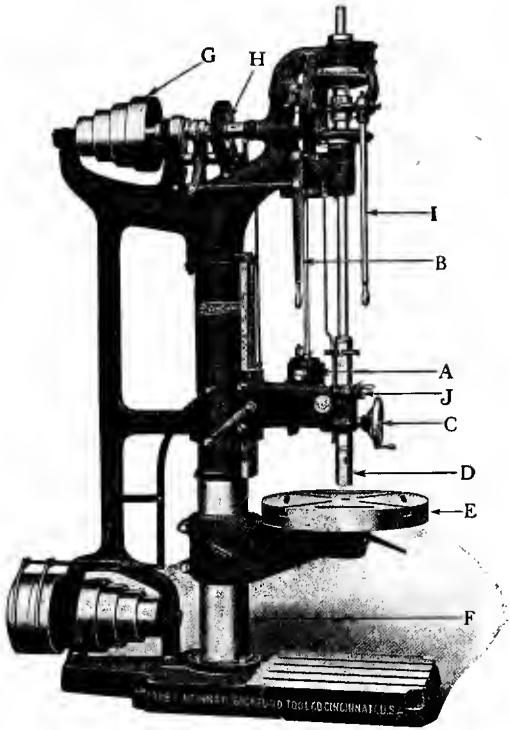
PART I

PART II

Part III

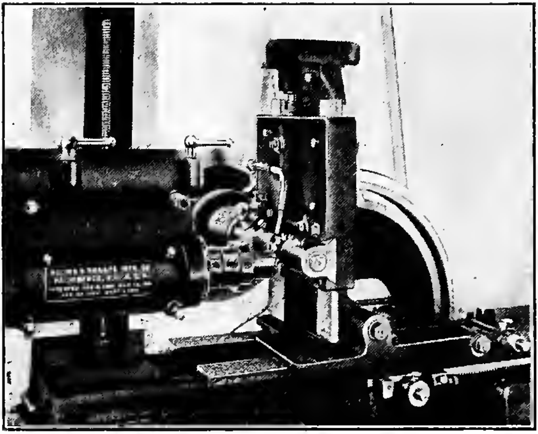
Part IV

Appendix

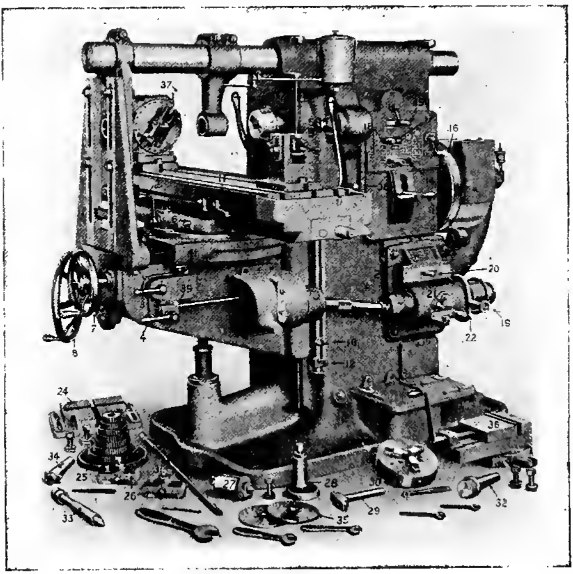


DRILLING MACHINE

A—Feed box.
B—Back-gear lever.
C—Hand feed wheel.
D—Spindle.
E—Work-table.
F—Column.
G—Cone pulley.
H—Back-gears.
I—Reverse lever.
J—Feed trip.



UNIVERSAL GEAR CUTTER



PLAIN MILLING MACHINE

PART I

Signs, Symbols and Abbreviations Used in Mathematics in General, with Illustrations of Their Uses

$=$ signifies equals, 7 and 2 = 9.

$+$ = plus, $7 + 2 = 9$

$-$ = minus, $9 - 7 = 2$.

$\times$ = times, $3 \times 2 = 6$.

$\div$ = divided by, $6 \div 2 = 3$. Note the alternative usage, $6/2 = 3$.

$\therefore$ = therefore, if $2 + 2 = 4 \therefore 4 - 2 = 2$.

$:$ = is to, $2 : 4$ as $3 : 6$.

$::$ = as, $2 : 4 :: 3 : 6$.

$>$ = greater than, $7 > 3$.

$<$ = less than $3 < 7$.

$\sqrt{\ }$ or $\sqrt[2]{\ }$ = radical sign or square root, $\sqrt{4} = 2$, or $\sqrt[2]{4} = 2$.

$\sqrt[3]{\ }$ = cube root, $\sqrt[3]{8} = 2$.

$\sqrt[5]{\ }$ = fifth root, $\sqrt[5]{32} = 2$.

4^2 = four squared (second power of 4) $4^2 = 4 \times 4 = 16$.

4^3 = four cubed (3d power of 4) $4^3 = 4 \times 4 \times 4 = 64$.

log = logarithm of, $\log 2056 = 3.3131$.

π = pi or 3.1416 or (3.14159265359) or approximately $3\frac{1}{7}$, $2 \times \pi = 6.2832$.

g = acceleration of gravity (32.16 ft. per sec. per sec.), $2 \times g = 64.32$.

tan = tangent of, $\tan 20° = 0.36397$.

cot = cotangent of, $\cot 20° = 2.7475$.

sin = sine of, $\sin 20° = 0.34202$.

cos = cosine of, $\cos 20° = 0.93969$.

$\overline{\quad}$ = vinculum, $2 \times \overline{4 + 2} = 2 \times 6 = 12$.

( ) = parentheses, $2 \times (3 + 2) \div 5 = 2 \times 5 \div 5 = 2$.

{ } braces, $= 24 \div \{2 \times (4 + 2)\} = 2.$

[ ] = brackets, $[3 \times \{2 \times (4 + 2)\}] \times 2 = 72.$

These signs (vinculum, parentheses, braces and brackets) denote that the quantities included within them are to be treated as a whole.

$\angle$ = angle, The $\angle$ of the U.S.S. thread is 60°.

∟ = right angle, The wall is at ∟ to the floor.

$\perp$ = perpendicular, The pole stands $\perp$ to the base.

$||$ = parallel, The shafts of two spur gears are $||$.

° = degree (circular arc or thermometer). The temperature was 80° to-day, or It is a 30° angle.

′ = minutes or feet, It moved 6′ in 30′ time.

″ = seconds or inches, It moved 3.125″ in 5″ time.

h.p. = horse power, The engine develops 25 h.p.

b.h.p. = brake horse power, That gas engine develops 12 b.h.p.

i.h.p. = indicated horse power, 15 is the actual h.p. developed in the cylinder.

B.t.u. = British thermal unit, That lb. of coal contains 14000 B.t.u.

kw. = kilowatt or 1000 watts, The electric generator develops 90 kw.

m.e.p. = mean effective pressure, The m.e.p. is about 80 lbs.

r.p.m. = revolutions per minute, The pulley runs 300 r.p.m.

f.p.m. = feet per minute, The piston travels 1500 f.p.m.

Notation and Numeration

Arithmetic is the science and application of numbers.

A **Number** is a unit or collection of units.

An **Integer** or whole number is composed of whole units only.

A **Concrete Number** is one applied to any particular thing. Example: 5 boys, 6 men, 25 bolts.

An **Abstract Number** is one not applied to any particular thing. Example: 5, 6, 25.

Notation is the art of expressing number by figures or letters. Example: XII = 12, 5 = five.

Numeration is the art of reading numbers which have been expressed. Example: 545 is read—five hundred forty five.

The **Arabic Notation** is the method of expressing numbers by figures. Example: 0 = naught or cypher, 1 = one, 2 = two.

The location of a figure in a number denotes its value. Example: 2 alone = two, but in 21 the value of the 2 is increased by its location.

In reading figures, the first one on the right is units and then in order come, tens of units, hundreds of units, thousands, tens of thousands, hundreds of thousands, millions, tens of millions, hundreds of millions, billions, tens of billions, hundreds of billions, trillions, tens of trillions, hundreds of trillions, etc. Example: 845624891623125.

It is customary to separate numbers into three figure groups by commas. Example: 845,624,891,623,125.

Rule for Numeration or Reading.—Begin at the left and read each period containing one or more figures as if it stood alone, adding its name: thus, the above number is read 845 trillion, 624 billion, 891 million, 623 thousand, 125.

EXERCISES

1. Write a seven figure number.
2. Write an integer.
3. Give an example of a concrete number.
4. Give an example of an abstract number.
5. Express by Arabic notation: eighty-five.
6. Write out in words: 846,762,845,967,843.
7. Why does the 9 in 95 represent a greater value than the 9 in 59?
8. Separate into periods: 1648432.

Addition

Addition, subtraction, multiplication and division are the most important processes necessary in mathematical calculation.

Addition is the process of finding the sum of two or more numbers. The sign + indicates that addition is to be performed and is read plus. Example: $2 + 5 = 7$.

The sign of equality is = and is used thus: $2 + 5 = 7$.

In adding concrete numbers, they must be of the same denomination.

Rule.—In adding, place the numbers in a vertical column with units under units, tens under tens, etc. Place the right hand figure of the sum of the right hand column under units, and if there be two figures in this sum, the left hand one is carried and added to tens column. Example:

$$\begin{array}{r} 843 \\ 246 \\ \underline{172} \\ 1261 \end{array} = \text{(Ans.)}$$

Subtraction

Subtraction is the process of taking one number from another. Example: $12 - 5 = 7$.

The **Minuend** is the number from which the other is to be taken. Example: $12 - 5 = 7$; 12 is the minuend.

The **Subtrahend** is the number which is to be taken from the minuend. Example: $12 - 5 = 7$; 5 is the subtrahend.

The **Remainder** is what is left after the subtrahend has been taken from the minuend. Example: $12 - 5 = 7$; 7 is the remainder.

The (−) sign (called minus) indicates that subtraction is to be performed and that the number following it is to be taken from the number before it.

Rule.—In subtraction, the units must fall under units, the tens under tens, etc. Subtract the right hand figure of the subtrahend from the right hand figure of the minuend and place the remainder directly below. If necessary, borrow of the next left hand figure of the minuend to make this possible. Then subtract from the remaining minuend next. The sum of the subtrahend and remainder should always be equal to the minuend. Example:

$$
\begin{array}{rrl}
 & 4625 & \text{minuend} \\
 & -\ \underline{3287} & \text{subtrahend} \\
 & 1338 & \text{remainder} \\
\text{Proof:} & +\ \underline{3287} & \text{subtrahend} \\
 & =\ 4625 & \text{minuend}
\end{array}
$$

Multiplication

Multiplication is the process of taking or increasing one number a certain numbers of times.

The **Multiplier** is the number which shows how many times the other number is to be taken. Example: $5 \times 8 = 40$; 5 is the multiplier.

The **Multiplicand** is the number which is to be taken a certain number of times. Example: $5 \times 8 = 40$; 8 is the multiplicand.

The **Product** is the result obtained by taking one number a certain number of times. Example: $5 \times 8 = 40$; 40 is the product.

The $\times$ sign (called times) indicates that multiplication is to be performed and that the number following is to be taken as many times as there are units in the number before. Example: $5 \times 8 = 40$.

Rule.—In multiplication, the units in the multiplier are placed under the units in the multiplicand, and the right hand figure of the product placed directly below the other

right hand figures. Each figure of the multiplicand, beginning at the right, is multiplied by each figure of the multiplier and the right hand figure of each partial product is placed in turn directly under the figure used as multiplier. Partial products are placed on different lines. The sum of the partial products will equal the required product.

$$\begin{array}{lrl} \text{Example:} & 842 & \text{multiplicand} \\ & \times\ \underline{245} & \text{multiplier} \\ & 4210 & \\ & 3368 & \\ & \underline{1684} & \\ & 206290 & \text{product} \end{array}$$

Division

Division is the process of determining how many times one number is contained in another. Example: 12 ÷ 4 = 3, 12/4 = 3.

The **Dividend** is the number to be divided. Example: 12 ÷ 4 = 3; 12 is the dividend.

The **Divisor** is the number by which we divide. Example: 12 ÷ 4 = 3; 4 is the divisor.

The **Quotient** shows how many times the divisor is contained in the dividend. Example: 12 ÷ 4 = 3; 3 is the quotient.

The ÷ sign indicates that division is to be performed and that the number before the sign is to be divided by the number following the sign.

Short Division is used where the divisor contains but one figure. Example: 144 ÷ 8 = 18, 144/8 = 18.

Long Division is used where the divisor contains more than one figure. Example: 378 ÷ 14 = 27.

Rule: Short Division.—Place the divisor at the left of the dividend separated by a line and draw a line under the

dividend. Try the divisor into the first or first two figures of the dividend, as is necessary, and place the quotient under the line. If the divisor does not go an even number of times, the remainder is prefixed to the next figure in the dividend and the process repeated. If there is a final remainder, the answer becomes a mixed number.

Example: $29757 \div 7 = 4251$. Solution:

$$\begin{array}{r|l} 7 & \underline{29757} \\ & 4251 \end{array}$$

Rule: Long Division.—Place the divisor at the left of the dividend, separated by a line, and place the quotient either above or to the right of the dividend. Try the divisor into the first group of figures which gives a number larger than the divisor, place the first figure of the quotient above the dividend and (after multiplying it by the divisor) place this product below the figures divided into and subtract. The remainder prefixed to the next figure brought down, forms the new trial dividend. Repeat until all figures of the dividend are brought down.

Example 1, $1041741 \div 243 = 4287$.

Solution: 4287 = (Ans.)

$$\begin{array}{r|r} & 4287 \\ \cline{2-2} 243 & 1041741 \\ & \underline{972} \\ & 697 \\ & \underline{486} \\ & 2114 \\ & \underline{1944} \\ & 1701 \\ & 1701 \end{array}$$

Example 2, $978804 \div 243 = 4028$.

Solution:

$$
\begin{array}{r|l}
 & 4028 \\
\cline{2-2}
243 & 978804 \\
 & \underline{972} \\
 & 68 \\
 & \underline{00} \\
 & 680 \\
 & \underline{486} \\
 & 1944 \\
 & 1944
\end{array}
$$

EXERCISES

1.	(a)	Add	842658, 36541, 274, 896
	(b)	"	4657, 125, 84
2.	(a)	Add	32689, 54, 97, 125, 18
	(b)	"	63, 762, 81, 91, 5000
3.	(a)	Add	666, 83, 12, 6, 7
	(b)	"	5589, 664, 98, 54376
4.	(a)	Subtract	864 from 86945
	(b)	"	123 " 465
5.	(a)	Subtract	64 from 99
	(b)	"	126 " 465
6.	(a)	Subtract	10 from 10000
	(b)	"	986 " 2245
7.	(a)	Multiply	84 by 84
	(b)	"	123 " 17
8.	(a)	Multiply	164 by 140
	(b)	"	14000 " 7
9.	(a)	Multiply	16845 by 93
	(b)	"	2345 " 3456
10.	(a)	Divide	20232 by 24
	(b)	"	8262 " 51
11.	(a)	Divide	20539 by 893
	(b)	"	6880635 " 107
12.	(a)	Divide	140010 by 13
	(b)	"	5001613 " 4957

Cancellation

Cancellation is a short method for performing multiplication and division, applicable in some cases.

Rule.—In cancellation, all the dividends are placed above the line, all the divisors below, and common factors from each taken out before multiplying. Example: Divide the product of 8, 16 and 12 by the product of 8, 4 and 3.

Solution:
$$\frac{\cancel{8} \times \overset{4}{\cancel{16}} \times \overset{4}{\cancel{12}}}{\cancel{8} \times \cancel{4} \times \cancel{3}} = 4 \times 4 = 16. \quad \text{(Ans.)}$$

Least Common Multiples

A **Multiple** of a number is a quantity which can be divided evenly by the number.

The **Least Common Multiple** of two or more numbers is the least number which can be divided evenly by both of them.

Rule.—The least common multiple of two or more numbers is found by dividing each number into its smallest factors and then taking each factor the greatest number of times it is found in any one number. Example: L.C.M. of 12 and 15.

Solution:
$$\begin{array}{l} 12 = 2 \times 2 \times 3 \\ \underline{15 = 3 \times 5 \qquad\qquad} \\ \quad 2 \times 2 \times 3 \times 5 = 60 \text{ L.C.M.} \quad \text{(Ans.)} \end{array}$$

EXERCISES

1. Divide the product of 48, 39 and 25 by the product of 5, 6 and 13.
2. Divide the product of 45, 64 and 49 by the product of 7, 9 and 8.
3. $(40 \times 150 \times 10) \div (100 \times 25) = ?$
4. (*a*) Find L.C.M. of 18, 14 and 12.
   (*b*) " " " 12, 16 and 4.
5. (*a*) Find L.C.M. of 3, 4 and 5.
   (*b*) " " " 81 and 9.

6. (*a*) Find L.C.M. of 72, 3, 9, 2 and 6.
   (*b*) " " " 9 and 10.
7. (*a*) Find L.C.M. of 1, 2, 3, 4 and 5.
   (*b*) " " " 2, 6, 9, 5, 4 and 3.

Common Fractions

A **Fraction** of any thing is a part of it. Example: $\frac{1}{2}$ = one half or $\frac{2}{3}$ = two thirds.

The **Denominator** of a fraction is the number below the line and it indicates into how many equal parts the thing is to be divided. Example: $\frac{5}{8}$; 8 is the denominator.

The **Numerator** is the number above the line and it indicates how many of the equal parts are to be taken. Example: $\frac{1}{2}$; 1 is the numerator.

A **Common Fraction** is one whose denominator is not 10, 100, 1000, etc. Example: $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{5}$.

A **Proper Fraction** is one whose numerator is less than the denominator. Example: $\frac{2}{3}$.

An **Improper Fraction** is one whose numerator is greater than the denominator. Example: $\frac{3}{2}$.

A **Simple Fraction** is one not connected with another. Example: $\frac{2}{3}$.

A **Compound Fraction** is a fraction of a fraction. Example, $\frac{2}{3}$ of $\frac{3}{4}$.

A **Complex Fraction** is one having a fraction in both or at least one of its terms. Example: $\dfrac{\frac{2}{3}}{3}$ or $\dfrac{\frac{1}{2}}{\frac{2}{3}}$.

The line drawn between the numerator and denominator of a fraction indicates division, or that the numerator is to be divided by the denominator. Example: $\frac{3}{4} = 3 \div 4$.

A **Mixed Number** consists of a whole number and a fraction written together. Example: $3\frac{1}{2}$ or $12\frac{1}{4}$.

Rule.—To reduce a mixed number to an improper fraction, multiply the whole number by the denominator and add

the numerator, so as to form a new numerator. Example: $3\frac{1}{2} = \frac{7}{2}$.

Rule.—To reduce an improper fraction to a mixed number, divide the numerator by the denominator. The quotient will be the whole number and the remainder placed over the divisor will be the fraction. Example: $\frac{7}{2} = 3\frac{1}{2}$. Solution: $7 \div 2 = 3$ and 1 remainder.

Rule.—To reduce a whole number to a fraction, simply multiply the whole number by the denominator of the fraction desired. Example: 5 to 4ths $= \frac{5 \times 4}{4} = \frac{20}{4}$.

Rule.—The value of a fraction is not changed by multiplying or dividing both terms by the same number.

$$\text{Example: } \frac{2 \times 2}{3 \times 2} = \frac{4}{6} \text{ or } \frac{2}{3}. \qquad \frac{4 \div 2}{6 \div 2} = \frac{2}{3} \text{ or } \frac{4}{6}.$$

Addition of Fractions

Rule.—In adding fractions, the least common multiple of all of the denominators must first be determined, and then all fractions changed in form to this denominator.

Example: Add $\frac{2}{3}, \frac{3}{4}$ and $\frac{5}{6}$ L.C.M. of 3, 4 and 6 = 12. $\frac{2}{3} = \frac{8}{12}$, $\frac{3}{4} = \frac{9}{12}$, $\frac{5}{6} = \frac{10}{12}$. $\frac{8}{12} + \frac{9}{12} + \frac{10}{12} = \frac{27}{12} = 2\frac{3}{12}$ or $2\frac{1}{4}$ (Ans.).

Where mixed numbers are to be added, the whole numbers are added separate and then added to the result. Example:

$$2\frac{1}{2} + 3\frac{1}{4} + \frac{1}{3} = 2\frac{6}{12} + 3\frac{3}{12} + \frac{4}{12} = 5\frac{13}{12} \text{ or } 6\frac{1}{12} \text{ (Ans.).}$$

Rule.—To reduce a fraction to higher terms, divide the

required denominator by the denominator of the fraction, and multiply this quotient by the numerator. Example: Reduce $\frac{1}{2}$ to 8ths; $8 \div 2 = 4$. $4 \times 1 = 4$, or $\frac{1}{2} = \frac{4}{8}$ (Ans.).

Subtraction of Fractions

In subtracting fractions, the denominators must be reduced to a common denominator or L.C.M. Example:

$\frac{2}{3} - \frac{1}{2} = \frac{4}{6} - \frac{3}{6} = \frac{1}{6}$ (Ans.).

In subtracting fractions where the minuend is a mixed number with a fraction smaller than the subtrahend, the minuend can then be reduced to an improper fraction before subtracting. Example:

$$2\frac{1}{2} - \frac{2}{3} = \frac{5}{2} - \frac{2}{3} = 1\frac{5}{6} - \frac{4}{6} = \frac{11}{6} \text{ or } 1\frac{5}{6} \text{ (Ans.).}$$

Multiplication of Fractions

In multiplication of fractions, reduction to common denominators is not necessary. Example: $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$ (Ans.).

Rule.—To multiply a fraction by a whole number, multiply the numerator or divide the denominator by the whole number. Example: $\frac{4}{5} \times 5 = \frac{20}{5} = 4$ (Ans.).

Rule.—To mult'p y two or more fract ons together, multiply all the numerators for a new numerator and all the denominators for a new denominator and then reduce to lowest terms. Example:

$\frac{1}{2} \times \frac{3}{4} \times \frac{1}{5} = \frac{3}{40}$ (Ans.); $\frac{1}{3} \times \frac{2}{8} \times \frac{4}{9} = \frac{8}{216} = \frac{1}{27}$ (Ans.).

Rule.—To multiply two or more fractions and mixed

numbers together, the mixed number can first be reduced to improper fractions and then the preceding rule be used.

Example: $1\frac{1}{2} \times 2\frac{1}{4} = \frac{3}{2} \times \frac{9}{4} = \frac{27}{8}$ or $3\frac{3}{8}$ (Ans.).

Rule.—To multiply fractions by cancellation, first reduce any mixed numbers to improper fractions and then cancel any factors which may be found in both the numerator and denominator. Example: $\frac{1}{\cancel{2}} \times \frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{3}}{5} = \frac{1}{5}$ (Ans.).

Division of Fractions

In the division of fractions, reduction to a common denominator is not necessary.

Rule.—To divide one fraction by another, first reduce any mixed numbers to improper fract ons, th n nv rt the divisor and proceed as in multiplication. Example:

$$\frac{3}{4} \div \frac{1}{4} = \frac{3}{4} \times \frac{4}{1} = \frac{12}{4} = 3, \quad \text{also} \quad \frac{3}{\cancel{4}} \times \frac{\cancel{4}}{1} = \frac{3}{1} = 3 \text{ (Ans.).}$$

EXERCISES

1. Reduce to mixed numbers: $\frac{80}{3}$, $\frac{22}{7}$, $\frac{17}{2}$, $\frac{19}{3}$, $\frac{109}{4}$, $\frac{172}{83}$, $\frac{126}{5}$, $\frac{894}{763}$, $\frac{123}{121}$, $\frac{13}{12}$.

2. Reduce to improper fractions: $5\frac{1}{4}$, $8\frac{1}{9}$, $6\frac{2}{3}$, $12\frac{7}{8}$, $14\frac{1}{9}$, $6\frac{7}{8}$, $11\frac{11}{12}$, $49\frac{48}{49}$, $888\frac{888}{889}$, $125\frac{9}{17}$.

3. Reduce to simple fractions: $\frac{1}{3}$ of $\frac{1}{2}$, $\frac{2}{5}$ of $\frac{9}{10}$, $\frac{1}{16}$ of $\frac{8}{9}$, $\frac{2}{7}$ of $\frac{14}{15}$, $\frac{3}{4}$ of $\frac{5}{9}$, $\frac{2}{3}$ of $\frac{9}{16}$, $\frac{1}{2}$ of $\frac{5}{4}$, $\frac{8}{17}$ of $\frac{5}{19}$, $\frac{121}{146}$ of $\frac{389}{463}$, $\frac{862}{143}$ of $\frac{1}{2}$.

4. Reduce to simple fractions:

$$\frac{\frac{62}{63}}{\frac{1}{2}}, \quad \frac{\frac{125}{132}}{\frac{8}{9}}, \quad \frac{\frac{18}{19}}{\frac{8}{9}}, \quad \frac{\frac{14}{15}}{\frac{15}{16}}, \quad \frac{\frac{1}{2}}{\frac{8}{9}}, \quad \frac{\frac{1}{4}}{5}, \quad \frac{\frac{1}{16}}{\frac{1}{4}}, \quad \frac{\frac{13}{15}}{\frac{13}{14}}, \quad \frac{\frac{8}{9}}{\frac{2}{3}}.$$

5. Reduce to fractions: 7 to 4ths, 18 to halves, 12 to 3rds, 8 to 12ths, 5 to 10ths, 6 to 3rds, 4 to 8ths, 6 to 7ths, 4 to 5ths, 17 to 13ths.

6. Reduce to lowest terms: $\frac{12}{14}$, $\frac{4}{12}$, $\frac{124}{128}$, $\frac{63}{9}$, $14\frac{5}{20}$, $18\frac{9}{18}$, $124\frac{120}{148}$, $5\frac{12}{1728}$, $14\frac{3}{7}$.

7. Reduce to higher terms: $\frac{5}{8}$ to 64ths, $\frac{2}{3}$ to 48ths, $\frac{3}{32}$ to 128ths, $\frac{15}{18}$ to 64ths, $\frac{13}{32}$ to 160ths, $1\frac{1}{2}$ to 8ths, $8\frac{1}{3}$ to 24ths, $\frac{13}{2}$ to 6ths.

8. Reduce to least common denominator: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{3}{8}$: also $\frac{8}{9}$, $\frac{3}{4}$, $\frac{2}{5}$: also $8\frac{1}{4}$, $3\frac{5}{8}$, $\frac{7}{8}$: also $\frac{1}{18}$, $5\frac{1}{3}$: also $\frac{8}{9}$, $\frac{1}{4}$, $2\frac{1}{2}$: also $5\frac{1}{8}$, $3\frac{1}{4}$, $\frac{1}{16}$:also $\frac{3}{8}$, $5\frac{1}{64}$, $\frac{7}{16}$: also $8\frac{1}{4}$, $2\frac{1}{2}$, $\frac{9}{32}$: also $8\frac{1}{2}$, $\frac{9}{64}$.

9. (a) Add $\frac{1}{2}$, $\frac{2}{3}$, $2\frac{1}{4}$, $5\frac{3}{8}$.
   (b) " $2\frac{5}{8}$, $25\frac{1}{2}$, $18\frac{3}{4}$, $1\frac{2}{3}$.

10. (a) Add 865, $\frac{1}{3}$, $\frac{1}{8}$.
   (b) " $41\frac{5}{13}$, $41\frac{5}{14}$.

11. (a) Add $22\frac{1}{3}$, $22\frac{1}{4}$, $22\frac{1}{5}$, $22\frac{1}{7}$.
   (b) " $21\frac{1}{3}$, $489\frac{1}{16}$, $2\frac{3}{8}$.

12. (a) Subtract $32\frac{5}{8}$ from 40.
   (b) " $18\frac{1}{2}$ " $19\frac{1}{3}$.

13. (a) Subtract $22\frac{5}{8}$ from $29\frac{1}{3}$.
   (b) " $2\frac{9}{64}$ " $3\frac{3}{32}$.

14. (a) Subtract $465\frac{5}{13}$ from $491\frac{1}{3}$.
   (b) " $100\frac{3}{4}$ " $9862\frac{5}{8}$.

15. (a) Multiply $\frac{1}{3}$ by $\frac{4}{5}$.
   (b) " $\frac{3}{4} \times 2\frac{1}{2} \times 5\frac{1}{4} \times \frac{5}{21}$.

16. (a) Multiply $\frac{2}{5}$ by $\frac{5}{8}$.
   (b) " $2\frac{1}{16} \times 4\frac{3}{32} \times 5\frac{1}{4} \times 4\frac{1}{8}$.

17. (a) Multiply $4\frac{5}{8} \times 1\frac{17}{32} \times \frac{8}{9} \times 2\frac{1}{4}$.
   (b) " $5\frac{1}{4} \times 19\frac{1}{8} \times 5\frac{2}{7} \times 8\frac{4}{5}$.

18. (a) Divide $\frac{2}{3}$ by $\frac{4}{7}$.
   (b) " $2\frac{1}{2}$ by $1\frac{1}{4}$.

19. (a) Divide $1\frac{1}{4}$ by $2\frac{1}{2}$.
   (b) " $84\frac{1}{3}$ by $17\frac{5}{9}$.

20. (a) Divide $265\frac{1}{8}$ by $4\frac{3}{8}$.
   (b) " $489\frac{1}{4}$ by $93\frac{2}{5}$.

21. If a machinist finds five different sizes on a shaft, each one being respectively $4\frac{1}{8}''$, $3\frac{1}{2}''$, $\frac{1}{4}''$, $\frac{5}{8}''$ and $\frac{7}{32}''$ long, how long is the entire shaft in inches?

22. What is the combined height of six blocks of steel placed upon each other, the sizes of each being $\frac{1}{4}''$, $\frac{3}{8}''$, $\frac{7}{64}''$, $\frac{1}{2}''$, $\frac{3}{16}''$ and $1\frac{1}{4}''$?

23. How many hours will it take to do all the operations on 5 shafts, if the centering on each requires $\frac{1}{4}$ hour, the rough turning $\frac{1}{2}$ hour, the finish turning $\frac{1}{3}$ hour and the threading $\frac{1}{6}$ hour?

24. How long will the front bearing on a lathe spindle be, if the entire length is 2 9/24 ft., the nose is $\frac{1}{4}$ ft., the space for the cone is 1 5/24 ft., the rear bearing is $\frac{1}{4}$ ft., and the remaining end is 7/24 ft.?

25. What will it cost to produce 12 gears at 30 cents per hour for labor if the first operation requires $1\frac{1}{2}$ hours, the second $1\frac{3}{8}$ hours, and the third $2\frac{1}{2}$ hours?

26. If 5 hangers are used to support a line shaft $42\frac{1}{2}$ ft. long, what is the distance between the hangers?

27. A man purchased a 2/5 interest in a factory and after giving away $\frac{1}{3}$ of this, divided the remainder among his three sons. What part of the original factory does each own?

28. How many posts $8\frac{1}{4}$ ft. long can be cut from a tree $32\frac{5}{8}$ ft. high?

29. How many ties, if they are placed $2\frac{1}{4}$ ft. apart, will be required for $326\frac{1}{4}$ ft. of track?

30. If a pulley is $3\frac{1}{8}$ ft. in circumference, how many times will it revolve in causing a belt $28\frac{1}{8}$ ft. long, to make one complete travel of its length?

Decimal Fractions

A decimal number is one containing figures to the right of the units place, separated from the unit figure by a decimal point or period. Example: 10.2 is a decimal number.

The figures to the right of the decimal point are read, in order, as tenths, hundredths, thousandths, ten thousandths, etc., each figure having one-tenth the value of the left hand figure preceding. Example: 10.201 = ten and two tenths and no hundredths and one thousandth, or ten and two hundred one thousandths, or $10\frac{201}{1000}$.

In some cases, the decimal fraction method is not exactly accurate, but for most calculations, it is sufficiently accurate, and much more convenient than the use of common fractions.

The denominator of a decimal fraction may be determined by placing the figure 1 under the decimal point and a cipher under each figure of the decimal. Example:

$$\left.\begin{matrix} 0.465 \\ 1000 \end{matrix}\right\} = 465/1000. \qquad \left.\begin{matrix} 1.2 \\ 10 \end{matrix}\right\} = 1\ 2/10.$$

The figures at the left of the decimal point represent the whole number and those at the right of the point the fraction.

Example: $25.45 = 25\frac{45}{100}$.

Ciphers added at the right hand side of a decimal fraction do not change its value. Example: 3.25 or 3.250 or 3.2500.

A common fraction can be reduced to a decimal exactly or approximately by dividing the numerator by the denominator. Example: $\frac{5}{8} = 0.625$.

Rule.—To reduce a common fraction to a decimal, place a point at the right of the numerator and divide the denominator into this number. If it will not go, add a cipher to the right of the point and proceed as in division, adding ciphers as required. If after several divisions, the remainder does not disappear, it is probably a repeating decimal, and the remainder should be dropped. The quotient should be pointed off from the right as many places as the number of places in the dividend exceed the number in the divisor.

Example: $5/8 = 8\underline{)5.000}$ $\quad$ $5/6 = 6\underline{)5.000}$

$\quad 0.625$ (Ans.). $\quad 0.833$ (Ans.).

Moving the decimal point to the right, multiplies the number by (10) for each place moved. Moving to the left, divides by (10) for each place moved. Example:

$$3.45 \times 10 = 34.5 \text{ (Ans.)}. \quad 3.45 \div 10 = 0.345 \text{ (Ans.)}.$$

Addition of Decimals

In addition or subtraction of decimals, the points should always fall under each other in the fraction and in the sum.

Example:

$$\begin{array}{rl} & 2.12 \\ + & \underline{14.3} \\ & 16.42 \text{ (Ans.)}. \end{array}$$

Addition is performed the same as in the addition of whole numbers.

Subtraction of Decimals

Subtraction is performed the same as in the subtraction of whole numbers. Example:

$$\begin{array}{r} 34.620 \\ -\ \underline{4.315} \\ 30.305 \end{array} \text{ (Ans.).}$$

Multiplication of Decimals

In multiplication of decimals, the points are not required to fall under each other and the fractions are placed generally so that the right hand figures in the multiplier and multiplicand will fall under each other. Example:

$$3.45 \times 2.5 = \begin{array}{r} 3.45 \\ \underline{\times\ 2.5} \\ 1725 \\ \underline{690} \\ 8.625 \end{array} \text{ (Ans.).}$$

Rule.—In multiplication of decimal fractions, the number of places pointed off from the right in the product should equal the sum of the places in the multiplicand and multiplier. Example above.

Division of Decimals

Rule.—Division of decimal fractions is exactly the same as division of numbers, except by the adding of ciphers to the right of the decimal point in the dividend, which is sometimes necessary. Point off as many places in the quotient as the number of places in the dividend exceeds the number of places in the divisor. Example:

$$2.5 \div 1.25 = \underline{1.25}|2.50|\underline{2.0} \text{ (Ans.).}$$
$$2.50$$

To reduce a decimal fraction to a common fraction, place

the denominator of the decimal below it and reduce to lowest terms. Example:

$$0.625 = \frac{625}{1000} \text{ or } \frac{625 \div 125}{1000 \div 125} = \frac{5}{8} \text{ (Ans.).}$$

EXERCISES

1. (*a*) Add 3.25, 4.625, 46.865, 4.25.
   (*b*) " 86.000, 86000.3.
2. (*a*) Add 0.125, 125, 1.25.
   (*b*) " 4.65793, 46489.73.
3. (*a*) Add 2649.37, 0.0000467, 100.0009.
   (*b*) " 1004.373, 58.9876, 1.26578.
4. (*a*) Subtract 4.625 from 84.
   (*b*) " .86497 from 1.
5. (*a*) Subtract 3.460009 from 4.00.
   (*b*) " 0.006387 from 0.06487.
6. (*a*) Subtract 13.399 from 14.01.
   (*b*) " 133.99 from 407.387.
7. (*a*) Multiply 34.65 by 3.6.
   (*b*) " 287.03 by 48.0
8. (*a*) Multiply 465 by 1.5.
   (*b*) " 0.83 by 0.83.
9. (*a*) Multiply 0.625 by 0.75.
   (*b*) " 4.87 by 27.4.
10. (*a*) Divide 8.463 by 4.65 (to 3 places).
    (*b*) " 48.43 by 4.5 (to 3 places).
11. (*a*) Divide 165 by 0.165.
    (*b*) " 84003 by 0.0006.
12. (*a*) Divide 0.00006 by 3.
    (*b*) " 9.63 by 6.
13. (*a*) Reduce to decimal fractions: 1/64, 1/8.
    (*b*) " " " " 5/7, 3/8.
14. (*a*) Reduce to decimal fractions: 7/64, 8/9.
    (*b*) " " " " 2/3, 4 5/6.
15. (*a*) Reduce to common fractions: .5, .75.
    (*b*) " " " " 2.625, 4.40625.
16. (*a*) Reduce to common fractions: 3.546875, 7.1875.
    (*b*) " " " " 25.046875, 1.375.
17. What will 17.5 yards of canvas cost at \$0.43 per yard?

18. What will 25 castings, weighing 0.5 lbs. each, cost at $0.30 per lb.?

19. What rate must be charged for butter so that 12.5 lbs. will net $4.0625?

20. What will be the weight of 33.25 cu. in. of iron if it weighs 0.25 lbs. per cu. in.?

Percentage

Percentage means per hundred $\frac{1}{100} = 1\%$.

Percent means a certain number of hundredths.

The percent sign is (%) and represents hundredths.

Example: $10\% = \frac{10}{100}$ or 0.10 or 0.1.

The **Base** is the number on which the percent is calculated. Example: 5% of 12 = 0.6; 12 is the base.

The **Rate** percent is the number of hundredths of the base. Example: 5% of 12 = 0.6; 5% is the rate.

The **Percentage** is the result obtained by taking a certain number of hundredths of the base. Example: 5% of 12 = 0.6; 0.6 is the percentage.

The **Amount** is the sum of the percentage and base. Example: 5% of 12 = 0.6; 0.6 + 12 = 12.6, amount.

The **Difference** is the difference between the percentage and the base. Example: 5% of 12 = 0.6; 12 − 0.6 = 11.4, difference.

Rules: Base × rate = percentage.
Percentage ÷ rate = base.
Percentage ÷ base = rate.
Base × (1 + rate) = amount.
Amount ÷ (1 + rate) = base.
Base × (1 − rate) = difference.

EXERCISES

1. Find 3% of $800.00.
2. Find 45% of 200.
3. Find $7\frac{1}{2}\%$ of 20 gallons.
4. Find what percent 26 is of 326.
5. Find what percent 3 is of 2.
6. Find what percent 45 is of 45.

7. What number of gears must be ordered from foundry if 5% are poor castings, to have 250 good ones?

8. What number if increased by 8% of itself will give 400?

9. What amount of money must be invested at 5% to amount to $300 in one year's time?

10. If 30% is lost through friction, what power will be transmitted from a mechanical device if 24h.p. is supplied?

WEIGHTS AND MEASURES

Table of Linear Measure

12 inches	= 1 foot
3 feet or 36 inches	= 1 yard
$5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet	= 1 rod
40 rods	= 1 furlong
8 furlongs or 320 rods, or 1760 yards or 5280 feet	= 1 mile

Table of Square Measure

144 square inches	= 1 square foot
9 square feet	= 1 square yard
$30\frac{1}{4}$ square yards or $272\frac{1}{4}$ square feet	= 1 square rod
160 square rods or 4840 square yards or 43560 square feet	= 1 acre
640 acres	= 1 square mile

Table of Cubic Measure

1728 cubic inches	= 1 cubic foot
27 cubic feet	= 1 cubic yard
128 cubic feet	= 1 cord

Table of Avoirdupois

435.5 grains	= 1 ounce
16 ounces or 7000 grains	= 1 pound
100 pounds	= 1 hundredweight
20 cwt. or 2000 pounds	= 1 ton
2240 pounds	= 1 long ton

Table of Troy Weight

24 grains	= 1 pennyweight
20 pennyweights or 480 grains	= 1 ounce
12 ounces or 5760 grains	= 1 pound

Table of Apothecaries Weight

20 grains	= 1 scruple
3 scruples or 60 grains	= 1 dram
8 drams or 480 grains	= 1 ounce
12 ounces or 5760 grains	= 1 pound

The grain is the same in Avoirdupois, Troy and Apothecaries weights.

Table of Liquid Measure

4 gills	= 1 pint
2 pints or 8 gills	= 1 quart
4 quarts or 231 cubic inches	= 1 U. S. gallon
$31\frac{1}{2}$ gallon	= 1 barrel
2 barrels	= 1 hogshead

Table of Dry Measure

2 pints	= 1 quart
8 quarts or 16 pints	= 1 peck
4 pecks or 2150.42 cubic inches	= 1 bushel

Table of Angular Measure

60 seconds	= 1 minute
60 minutes	= 1 degree
90 degrees	= 1 quadrant
360 degrees	= 1 circle

Table of Time Measure

60 seconds	= 1 minute
60 minutes or 3600 seconds	= 1 hour
24 hours or 1440 minutes	= 1 day
7 days	= 1 week
52 weeks or $365\frac{1}{4}$ days	= 1 year
100 years	= 1 century

Miscellaneous Table

1 cubic foot	= 7.48 gallons
1 inch	= 25.4 millimeters
231 cubic inches	= 1 U. S. gallon.
660 feet	= 1 furlong
3 miles	= 1 league
$2\frac{1}{2}$ feet	= 1 military pace

2 yards	= 1 fathom
$24\frac{3}{4}$ cubic feet	= 1 perch
2150.42 cu. in.	= 1 U. S. bushel
5760 grains	= 1 pound Troy or Apothecaries'
7000 grains	= 1 pound Avoirdupois
144 cu. in.	= 1 board foot
39.37 inches	= 1 meter
62.5 pounds	= 1 cubic foot of water at 62° F.
264.2 gallons	= 1 cubic meter
1550 sq. in.	= 1 sq. meter
3.168 grains	= 1 carat
15.432 grains	= 1 gram
746 watts	= 1 h.p.
1000 watts	= 1 k.w.
0.26 lbs. cast iron	= 1 cubic inch
0.283 lbs. steel	= 1 cubic inch
0.092 lbs. aluminum	= 1 cubic inch
0.300 lbs. brass	= 1 cubic inch
2545 B.t.u.	= 1 h.p. hour

Rule.—To change a quantity to a higher denomination, divide the quantity by the number of parts of the quantity required to make one of the higher denomination. Example: Change 15 inches to feet. $15 \div 12 = 1\frac{1}{4}$ ft. (Ans.).

Rule.—To change a quantity to a lower denomination, multiply the quantity by the number of parts of the lower denomination required to make one of the quantity. Example: Change 4 feet to inches. $4 \times 12 = 48$ inches (Ans.).

EXERCISES

1. Reduce 12 rods, 4 yards to inches.
2. Reduce 200 inches to higher denomination.
3. Reduce 17,000 feet to miles and feet.
4. Reduce 95 furlongs to miles and feet.
5. Reduce $17\frac{1}{2}$ yards to feet and inches.
6. Find area of field in acres which is 3 miles by 2 miles.
7. Find number of square inches of surface on a box $3'' \times 4'' \times 5''$.
8. Find number of sq. yds. and sq. ft. in tract of land 480×500 ft.

9. Reduce 1225 sq. yds. to sq. ft.
10. Reduce 127,050 sq. yds. to sq. rds.
11. What will be the cubic contents of a box 14″ × 92″ × 71″ in cu. ft.?
12. Reduce a cord of wood to cubic inches.
13. Reduce 88 cu. yds. to cu. ft.
14. Reduce 145 cu. ft. to cu. in.
15. How many grains in 14 lbs. 12 oz. (Avoirdupois)?
16. How many ounces in a ton?
17. How many tons and ounces in 3,000,000 ounces?
18. How many hundred weight in 5 tons?
19. Reduce to ounces, 3 tons, 4 cwt., 23 lbs.
20. Reduce to pwt., 400 grains.
21. Reduce to Troy ounces and grains, 50,000 grains.
22. Reduce to Troy pounds and ounces, 75 ounces.
23. Reduce to Troy ounces, 5 Troy pounds and 200 grains.
24. Reduce to grains, 200 Troy lbs.
25. Reduce to scruples, 200 grains.
26. Reduce to drams, 1500 grains.
27. Reduce to Apothecary lbs. and oz., 43,000 grains.
28. Reduce to Apothecary ounces, 7 lbs.
29. Reduce to Apothecary drams, 8 lbs. 3 oz.
30. Reduce to barrels, 252 gallons.
31. Reduce to quarts, 42 pints.
32. Reduce to gills, 2 barrels.
33. Reduce to bushels, 64 pints.
34. Reduce to pints, 64 bushels.
35. Reduce to gallons and gills, 180 gills.
36. Reduce to quarts, 18 pecks.
37. Reduce to pecks, 800 quarts.
38. Reduce to pints, 40 bushels.
39. Reduce to seconds, 90 degrees.
40. Reduce to degrees and minutes, 400 minutes.
41. Reduce to seconds, 80 degrees and 45 minutes.
42. Reduce to minutes, 3600 seconds.
43. Reduce to seconds, 360 degrees.
44. Reduce to days, 14 years.
45. Reduce to years, $3287\frac{1}{4}$ days.
46. Reduce to minutes, 45 hours.
47. Reduce to hours and minutes, 27 days.
48. Reduce to minutes, 2 years.

49. Reduce $4\frac{1}{8}''$ to millimeters.
50. Reduce 120 millimeters to inches.

Ratio and Proportion

Ratio is the comparative size of two numbers. Example: 12 to 8 = $12 \div 8 = 1\frac{1}{2}$.

The sign of ratio is a colon (:). Example: 8 : 4.

The first term of a ratio is the **Antecedent.**

The second term of a ratio is the **Consequent.**

The two terms of a ratio must be like numbers, as \$8 : \$4, 5 men to 3 men.

A **Compound Ratio** is the product of two or more simple ratios. Example: $\left\{\begin{matrix} 4 : 3 \\ 5 : 2 \end{matrix}\right\}$ as 10 : (?).

A compound ratio is reduced to a simple one by multiplying the antecedents together for a new antecedent and the consequents for a new consequent. Example:

$$\left\{\begin{matrix} 4 : 3 \\ 5 : 2 \end{matrix}\right\} : : 10 : (?) = 20 : 6 : : 10 : (?).$$

A **Proportion** is an equality of two ratios. Example: 12 is to 6 as 4 is to 2.

The sign of proportion is the double colon (: :). Example: 14 : 7 : : 4 : 2.

The first and fourth terms of a proportionate are called the **Extremes** and the second and third are called the **Means.** Example: 14 : 7 : : 4 : 2.
E M M E

In a **Direct Proportion** both couplets are direct ratios. Example: 5 men : 8 men : : \$30 : \$48. Where the amount 8 men would earn is required.

An **Inverse Proportion** requires reversing one of the couplets. Example 8 : 5 : : 30 days : 18 days. Where 30 days is the time required for 5 men to do a job and the time required for 8 men to do it is wanted.

When 3 numbers are in a proportion so that the 1st is to the 2d as the 2d is to the 3d, the 2d number is a **Mean Proportion** between the 1st and 3d. Example: 2–4–8.

Rules.—The product of means = product of extremes;
The product of mean ÷ extreme = other extreme;
The product of extremes ÷ mean = other mean.

The cause and effect method can be used to advantage in compound proportion by placing all the causes on one side of the : : and all the effects on the opposite side. Example: If 11 men can assemble 45 motors in 6 days of 10 hours each, how many men will it take to assemble 81 motors in 12 days of 11 hours each?

1st cause 2d cause 1st effect 2d effect

$$\left\{\begin{matrix}11\text{ men}\\ 6\text{ days}\\ 10\text{ hours}\end{matrix}\right\} : \left\{\begin{matrix}?\text{ men}\\ 12\text{ days}\\ 11\text{ hours}\end{matrix}\right\} :: 45\text{ motors} : 81\text{ motors.}$$

$$\frac{\text{extremes } 11 \times 6 \times 10 \times 81}{\text{means } ? \times 12 \times 11 \times 45} = 9\text{ men} = \text{(Ans.)}.$$

EXERCISES

1. If 3 men can drill 200 pieces in one hour, how many can 8 men drill?

2. If one gallon of oil will last a department 14 days of 10 hours each, how many gallons of oil will be used in 6 weeks of 55 hours each?

3. If the payroll each week is $500 for 15 men, at this ratio what would it be for 40 men?

4. If a Frankfort gas furnace consumes 5 ft. of gas per minute, when heating Novo Superior steel, what will it cost to operate the furnace $2\frac{1}{2}$ hours at 60 cents per 1000 ft.?

5. If we have a six pitch lead screw and a job of 10 pitch, what gear would be used with a 40 to cut this thread?

6. If 12 qts. of oil will last an auto owner 40 days, how long would it last four men?

7. If 48 bars of stock will last 8 screw machines 12 days, how long will they last 6 machines?

8. If it takes 20 days for 30 men to assemble a lot of automobiles, how long will it take 45 men to do it?

9. If 287,000 pieces a day can be produced by operating 7 dies at one time, how many days would be required to finish this many if 3 dies only were used?

10. If 5280 men can do a job in 73 days, how long will it take 427 men to do it?

11. If 40 men in 15 days, working 10 hours a day can make parts for 5 gas engines, how many men in 20 days working 8 hours a day would be required to make parts for 12 gas engines?

12. With tool steel at 25 cents a lb., it costs $180 a month to supply lathe tools in 3 departments and reamers in 20 departments. At this rate what will be the monthly cost if tool steel advances to 30 cents a lb. and three lathe and 8 drilling departments are added?

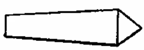
13. If it costs 30 cents to plate 240 pieces of sheet metal 4″ long and 2″ wide, what will it cost to plate 180 pieces 14″ long and $\frac{1}{2}$″ wide?

14. If 3 men working 6 hours a day can paint the side of a building 200 ft. long and 80 ft. high in 10 days, how long would it take 14 men working 8 hours a day to paint the side of a building 300 ft. long and 100 ft. high?

15. If a $\frac{1}{2}$″ piece of metal cut from the end of a 3″ diameter bar weighs 14 oz., what would be the weight of a $\frac{1}{2}$″ piece cut from a round bar of the same material but 8″ in diameter?

Taper Calculations

A piece is said to taper when there is a gradual and uniform increase or decrease in its diameter or thickness. Examples:

A lathe center  or a wedge  .

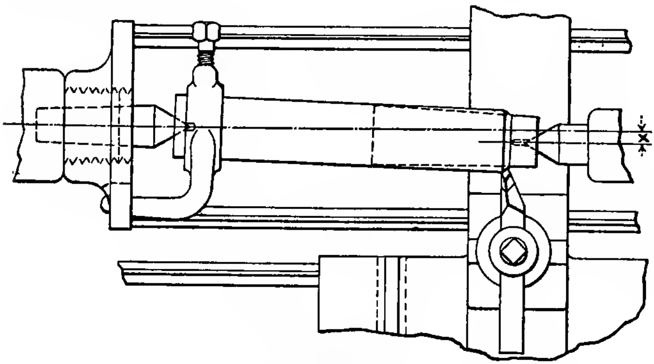
The **Amount of Taper** is expressed as a certain number of inches or parts of an inch per foot and indicates a variation in diameter or thickness of that amount in twelve inches of length.

The **Standard Tapers** used in shop work and their approximate tapers per foot are the Morse $\frac{5}{8}$″ per ft.; Brown & Sharpe $\frac{1}{2}$″ per ft.; Jarno 0.6″ per ft.; Sellers and Pipe taper $\frac{3}{4}$″ per ft.; Pratt & Whitney pins $\frac{1}{4}$″ per ft.

The taper on the B. & S. No. 10 is 0.516″ per ft., and on the Morse No. 1 to No. 6 it varies as much as 0.025 from $\frac{5}{8}$″ per ft.

Taper Turning on the lathe is sometimes accomplished by setting over the tail-stock and when this is done the taper per unit length as well as the length of the piece is required.

Rule.—Set the tail-stock over half the amount of the taper per ft. times the length of the work in feet. Example: To cut a Jarno taper on a piece 9″ long; Solution No. 1: $\frac{1}{2}$ of 0.6 = 3/10″; 3/10 × 3/4 = 9/40″ or 0.225″ set over = (Ans.); Solution No. 2: 0.6/2″ : 12″ : : (?) : 9″ = 0.3″ : 12″ : : 0.225″ : 9″.



EXERCISES

1. How far must the tail-stock of a lathe be set over to cut a B. & S. taper on piece $2\frac{1}{2}$″ long?

2. Jarno taper on piece 6″ long.
3. Morse No. 2 of 0.602″ on piece 2″ long.
4. Morse No. 4 of 0.623″ " " 5″ "
5. Sellers " " 8″ "
6. P. & W. pin " " 3″ "
7. Pipe tap " " 4″ "
8. Jarno " " 14″ "
9. B. & S. No. 10 " " $6\frac{1}{2}$″ "

10. If a taper piece measures 0.3875 at one point and 0.4365″ at a point $2\frac{1}{2}$″ away, how far should the tail stock be set over to cut this same taper on a piece 8″ long?

Interest

Interest is money paid for the use of money.

Percent means hundredths. Example: 6% = .06 = 6/100.

The **Rate of Interest** is the rate percent per annum of the principal paid for the use of money. Example: $100 at 6% for 3 years.—Rate of interest = 6%.

The sum loaned is called the **Principal.** Example: $100 at 6% for 3 years. $100 is the principal.

The principal plus the interest is called the **Amount.** Example: $100 at 6% simple interest for 3 years = $100 + $18 = $118 amount.

Simple Interest is interest on the principal only.

Annual Interest is simple interest upon the principal and upon any interest overdue.

Compound Interest is interest upon the principal and its unpaid interest combined at regular stated intervals. These intervals may be annual, semi-annual or quarterly.

Rule.—The **Simple Interest** on a sum of money is found by multiplying together the principal, the rate and the time in years. Example: $100 at 5% for $3\frac{1}{2}$ years.

$$
\begin{array}{rl}
\$100 & = \text{principal} \\
\underline{.05} & = \text{rate} \\
\$5.00 & = \text{interest for 1 year} \\
\underline{3\frac{1}{2}} & \\
\$17.50 & = \text{interest for } 3\frac{1}{2} \text{ years} = \text{(Ans.)}
\end{array}
$$

Rule.—The **Annual Interest** on a sum of money is found by adding together the interest on the principal for the entire length of time, and the interest on each year's interest for the time it is unpaid.

Example: $100 at 5% for $3\frac{1}{2}$ years.

1st year's interest = $5.

The interest for the first year remains unpaid for $2\frac{1}{2}$ years, the interest for the second year $1\frac{1}{2}$

years, and the interest for the third year $\frac{1}{2}$ years. Therefore, the unpaid interest draws interest for $2\frac{1}{2}$ years, $1\frac{1}{2}$ years and $\frac{1}{2}$ year, or $4\frac{1}{2}$ years, and the interest upon \$5.00 for that time is (\$5.00 at 5% for $4\frac{1}{2}$ years) = \$1.125.

The entire interest due is \$17.50 + \$1.125 = \$18.625 (Ans.).

Rule.—The amount at **Compound Interest** on a sum of money is found by adding the simple interest to the principal at regular stated intervals, and using this amount as a new principal. Example: \$100 at 5% for $3\frac{1}{2}$ years.

\$100	= principal.
5	= int. for 1st yr. at 5%.
105	= prin. for 2d yr.
5.25	= int. for 2d yr. at 5%.
110.25	= prin. for 3d yr.
5.51	= int. for 3d yr.
115.76	= prin. for 4th yr.
2.89	= int. for 6 mths.
118.65	= amt. for $3\frac{1}{2}$ yrs. at 5%.
100.00	= original principal.
18.65	= compound interest.

EXERCISES

1. Find simple interest on \$ 425 for 3 yrs. 4 mo. at 5%.
2. " " " " 1843 " 2 " 6 " " 6%.
3. " " " " 1500 " 5 " 9 " " 7%.
4. Find amount at annual interest on \$ 500 for $4\frac{1}{2}$ yrs. at 3%.
5. " " " " " " 1800 " 10 " " 6%.
6. " " " " " " 1250 " $3\frac{1}{2}$ " " 5%.
7. " " " compound " " 460 " 3 " " 4%.
8. " " " " " " 500 " $4\frac{1}{2}$ " " 3%.
9. " " " " " " 1800 " 10 " " 7%.
10. " " " " " " 800 " $7\frac{1}{2}$ " " 6%.

Pulley and Gear Diameters

Where power is transmitted from one shaft to another by means of pulleys, gears, belts, chains, etc., the ratio of the speeds is the inverse ratio of the diameter of the pulleys or gears: or in other words, the pulley diameters vary inversely as the speeds vary. Example: A line shaft (Fig. I) turning at 240 r.p.m. carries a pulley 12″ in diameter connected by a belt to a countershaft pulley 8″ in diameter; the proportion would read to find speed of 8″ pulley. 8″ : 12″ : : 240 r.p.m. : (?).

$$\frac{\overset{3}{\cancel{12}} \times \overset{120}{\cancel{240}}}{\underset{\cancel{2}}{\cancel{8}}} = 360 \text{ r.p.m. (Ans.).}$$

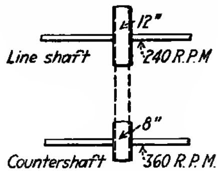


FIG. I

The above rule applies to simple gearing also providing the diameters are such as to give a whole number of teeth to the gears. Example: A gas engine crank shaft (Fig. II) turning 2000 r.p.m. has a 24 tooth gear keyed on to it which meshes with a 48 tooth gear on the cam shaft. Find the speed of cam shaft. 48 teeth : 24 teeth : : 2000 r.p.m. : (?).

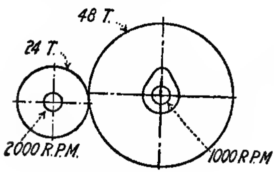


FIG. II

$$\frac{\overset{1000}{\cancel{2000}} \times \cancel{24}}{\underset{\cancel{2}}{\cancel{48}}} = 1000 \text{ r.p.m. (Ans.).}$$

The **Resultant Ratio** between the first and last shaft connected by compound gearing can be found by dividing the product of the number of teeth on all the driving gears by the product of all the driven gears. Example: Fig. III.

$$\frac{\text{Driving}}{\text{Driven}} = \frac{\overset{4}{\cancel{100}} \times \overset{6}{\cancel{120}} \times \overset{6}{\cancel{150}} \times \overset{2}{\cancel{100}}}{\cancel{25} \times \cancel{20} \times \cancel{25} \times \cancel{50}} = \frac{288}{1}.$$

∴ ratio is 288 to 1 (Ans.).

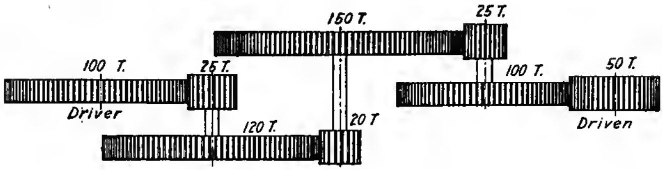


FIG. III

The above rule also applies to a train of shafts and pulleys connected by belts and in such cases the pulley diameters are used in place of the number of teeth as in gearing. Example Fig. IV.

$$\frac{\text{Drivers}}{\text{Driven}} = \frac{\overset{2}{\cancel{24}}'' \times \overset{2}{\cancel{20}}''}{\cancel{10}'' \times \cancel{12}''} = \frac{4}{1} \therefore \text{ratio is 4 to 1 (Ans.).}$$

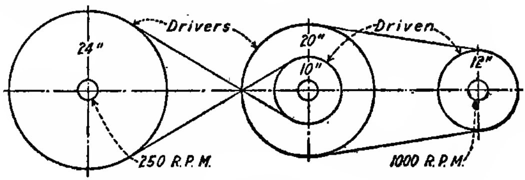


FIG. IV

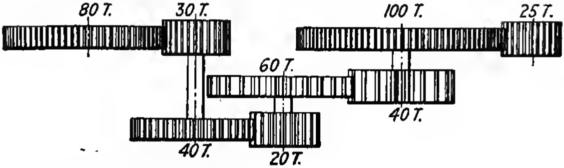
EXERCISES

1. If the front sprocket on a bicycle has 80 teeth and the back one 25 and the crank shaft is turned 80 r.p.m., how fast will the rear wheel go in r.p.m.?

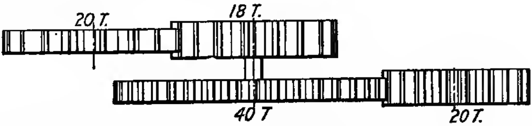
2. If the line shaft turns 280 r.p.m., and carries a 22″ pulley belted to a 10″ pulley, what will this run in r.p.m.?

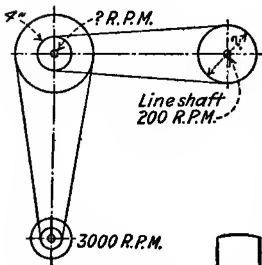
3. An electric motor running at 1500 r.p.m. has a 20 tooth gear on the armature shaft, running in mesh with a 180 tooth gear on the driven shaft. What is the speed in r.p.m. of the driven shaft.

4. Solve for ratio:



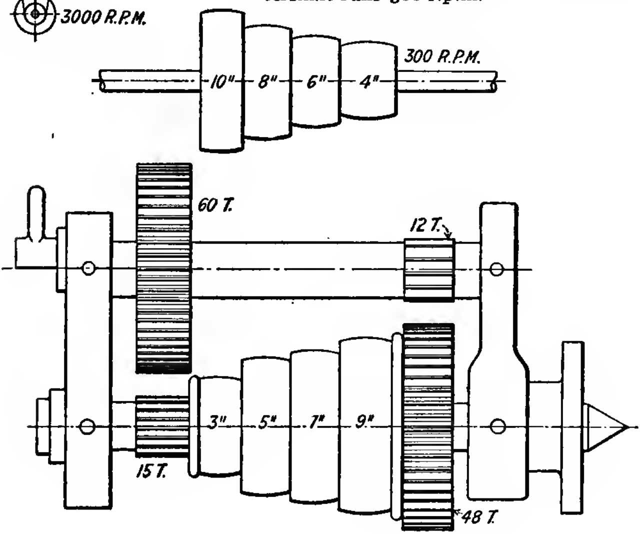
5. Solve for ratio:





6. What diameter pulley will be required on the line shaft to give 3000 r.p.m. to a 5″ emery wheel if the line shaft runs 200 r.p.m. the pulleys from countershaft to emery wheel spindle give a ratio of 1 to 10 and the pulley belted to line shaft is 4″ diameter?

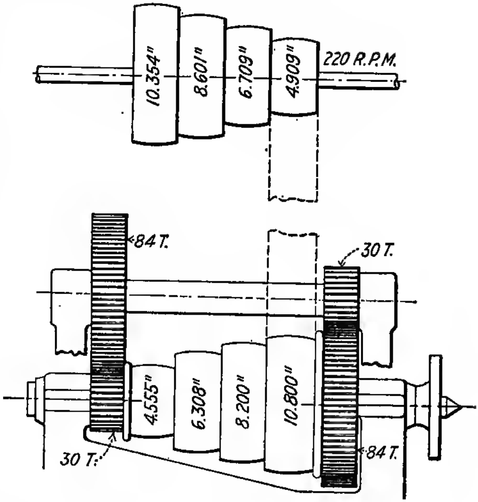
7. Calculate the eight different spindle speeds possible on the following 18″ back geared lathe, if the countershaft runs 300 r.p.m.

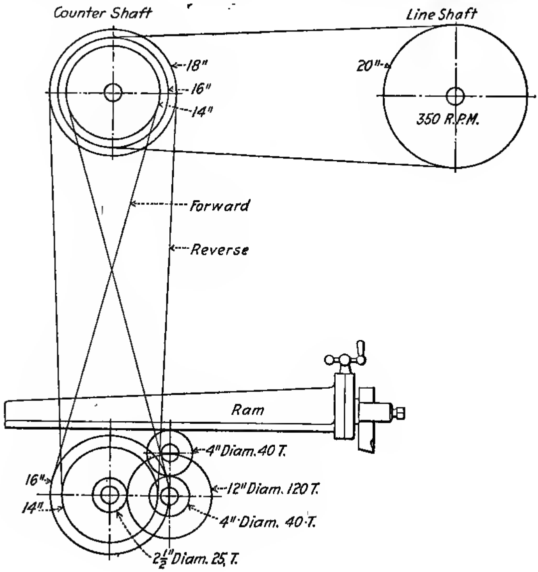


8. What will be the gear ratio in a train of gears if 6 driver gears of 21 teeth each drive 6 driven gears of 63 teeth each?

9. Calculate the eight different spindle speeds that will be obtained by the following sketch of a 16″ back gear lathe head.

10. What is the travel forward and reverse in feet per minute of the ram of a shaper, with pulleys and gears of the size given in the sketch?





Square Root, Involution and Evolution

The **Square Root** of a quantity is the number which when multiplied by itself will produce the quantity. Example: $\sqrt{16} = 4$, as $4 \times 4 = 16$.

When a number is multiplied by itself one or more times, the process is called **Involution.** The number thus multiplied is called the **Root** and the products are called the **Powers** of a number. Example: $4^2 = 16$ or $5^2 = 25$. This process is called involution.

Any number multiplied by itself once is said to be squared, and the product so obtained is called the **Square** or **Second Power** of the original number. Example: $3^2 = 9$. Thus 9 is the square or the second power of 3.

The small figure written near the top of a number, such as 3^2, 6^3 is called the **Index** or **Exponent** of the number.

Evolution is the reverse of involution and it means to extract or find the root of any given power. Example: $\sqrt{16} = 4$ or $\sqrt{25} = 5$. This process is called evolution.

The radical sign ($\sqrt{\ }$) indicates that the square root is to be extracted. Example: $\sqrt{16} = 4$.

If a radical be raised to the same power as the index of the radical, the radical sign is removed, thus $\sqrt[3]{2 \times 2 \times 2} = 2$ (Ans.).

In solving square root problems, the number must be first separated into two figure periods by starting from the decimal point and working both ways. As 4′83′92.84′6.

The index 2 need not be used, as $\sqrt[2]{843.25} = \sqrt{843.25}$.

Example: Extract the square root of 625. =

		2 5 (Ans.).
	2	$\sqrt{6'25.}$
	2	4
(T.D.)	40	2 25
	5	2 25
(R.D.)	45	

Example: Extract the square root of 119025. =

	3 4 5 (Ans.).
3	$\sqrt{11'90'25.}$
3	9
60	2 90
4	2 56
64	34 25
4	34 25
680	
5	
685	

Rule.—First point off in two figure periods from decimal point, then place square root of first period in answer and also at left of problem, next square first figure and subtract from period, also bring down next period. Next add first division to itself, annex cipher and label it trial divisor (T.D.). Next try trial divisor into dividend and place answer in answer above radical sign. Next add this number to trial divisor for real divisor (R.D.) and after multiplying, subtract from partial dividend, and bring down next period. Next add last number in answer to real divisor, annex one cipher and label it trial divisor and continue as before.

Exceptions to Rule.—(1) If trial divisor is found larger than dividend, annex one cipher to it and bring down next period in dividend. (2) If trial divisor will go a certain number of times, but real divisor will not, it then becomes necessary to use a smaller divisor.

If the quotient is placed above the periods, the decimal points will fall under each other.

EXERCISES

1. Extract the square root of 246016.
2. Extract the square root of 132496.

3. Extract the square root of 49284.
4. Extract the square root of 24649.
5. Extract the square root of 462.846743.
6. Extract the square root of 1234321.
7. Find the 9th power of 9.
8. Find the $\sqrt{6 \times 6 \times 6}$.
9. Find the 7th power of 7.
10. Find the 2d power of 125.

EXERCISES

A. Where *B* equals the base of a right angle triangle, *A* equals the altitude and *H* the hypotenuse, the rules $A^2 + B^2 = H^2$, $H^2 - B^2 = A^2$ and $H^2 - A^2 = B^2$ can be used for solving for one unknown side. (Fig. I.)

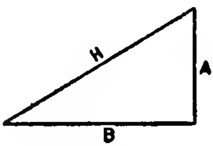


FIG. I

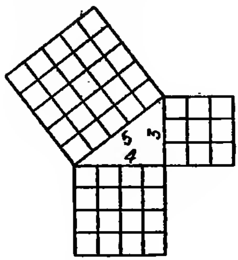


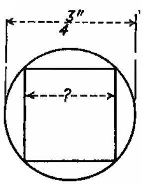
FIG. II

B. Fig. II shows diagramatically that a right angle triangle *B* of 4″, *A* of 3″ will give 5″ *H*.

1. How long a ladder will reach to the top of a fence 15 ft. high, if the ladder is placed 6 ft. away at the bottom?

2. How long must guy lines be to support a steel stack 125 ft. high if the lines are fastened 30 ft. from the top of the stack and the other ends are fastened to anchors in the ground, which are located 75 ft. from the base of the stack?

3. How much do I save on a corner by cutting across lots if the point where I leave the walk is 350 ft. from the corner and when I reach the walk again, it is 425 ft. from the corner?

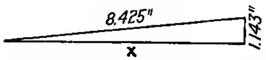


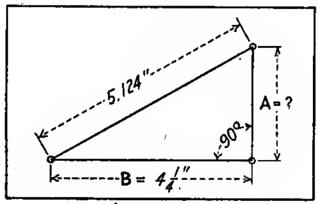
4. What is the largest size square steel stock that will fit into a $\frac{3}{4}$″ round hole?

5. What length guy will I have to purchase to fasten to the top of a mast 100 ft. high, and to the ground 150 ft. away?

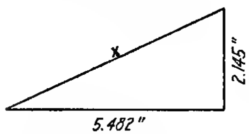
6. If in boring a jig the diagonal measures 5.124″ and the distance B $4\frac{1}{4}$″, what should the distance A measure?

7. Solve for (x):

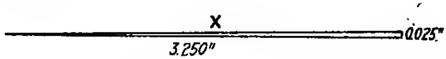




8. Solve for (x):



9. Solve for (x):



10. What size circle will just touch the corners of a rectangle 2″ × 3″?

11. What will be the volume of a wedge shaped piece of steel, as shown in problem No. 7, if the width is 2″?

12. What will be the greatest length rod that can be placed inside of a box 2 ft. wide × 3 ft. high × 4 ft. long?

13. What length piece of sheet metal which is 2 ft. wide can be placed inside of a box 2 ft. wide × 3 ft. high × 4 ft. long?

14. What length of wire will be required to wind around a bar 18″ long, 2″ in diameter with a $1\frac{1}{2}$″ lead?

Cube Root

The **Cube** or third power of a number is obtained if the number itself is repeated as a factor three times, thus the cube of 4 is $4 \times 4 \times 4 = 64$ and is written 4^3.

If a number is repeated 4 or 5 times, the product is the 4th or 5th power, thus $5^4 = 5 \times 5 \times 5 \times 5 = 625$.

The **Cube Root** of a given number is that number which, when repeated as a factor 3 times, will give a product equal to the given number, thus the cube root of 125 written $\sqrt[3]{125}$ equals 5, as $5 \times 5 \times 5 = 125$.

The 4th, 5th, etc. root of a given number are those numbers which, when repeated as factors 4, 5, etc. times, will give as a product the given number, thus $\sqrt[4]{625}$ equals 5, as $5 \times 5 \times 5 \times 5 = 625$.

Example: Extract the $\sqrt[3]{33076161.}$

$$
\begin{array}{ll}
3 \times 3 \times 3 = & 27 \\
3^2 \times 300 \quad = & 2700
\end{array}
$$

$$
\begin{array}{lrr}
(A) & 3^2 \times 300 \times 2 = & 5400 \\
(B) & 3 \times 30 \times 2^2 = & 360 \\
(C) & 2^3 = & 8 \\
\hline
 & & 5768
\end{array}
\qquad
\begin{array}{l|l}
33'076'161 & 132 \text{ (Ans.)} \\
27 & \\
\hline
60\ 76 & \\
\hline
57\ 68 & \\
\hline
3\ 08\ 161 & \\
3\ 08\ 161 &
\end{array}
$$

$$
\begin{array}{lrr}
(A') & 32^2 \times 300 = & 307200 \\
(B') & 32^2 \times 300 \times 1 = & 307200 \\
(C') & 32 \times 30 \times 1^2 = & 960 \\
 & 1^3 = & 1 \\
\hline
 & & 308161
\end{array}
$$

Rule.—To extract the cube root (assuming that the cube root of 33,076,161. is to be found) beginning at the unit figure or decimal point, point off the numbers in periods of 3 figures each.

Find the greatest whole number, the cube of which does not exceed the left hand period, and write this down as the first figure in the root (3).

Subtract the cube of 3 from the left hand period and bring down the next period of 3 figures and annex it to the remainder.

Multiply the square of the figure (3) in the root by the constant 300 $(3^2 \times 300) = 2700$, and find out how many times this number is contained in the number 6076.

This gives us a trial figure for the second figure of the root.

Next subtract from 6076 the sum of the following products:

1st—The square of the figure or figures already obtained in the root, except the last one, multiplied by 300 and this product multiplied by the figure just obtained in the root: (A) $3^2 \times 300 \times 2 = 5400$.

2d—The figure or figures already obtained in the root, except the last one, multiplied by 30, and this product multiplied by the square of the last figure obtained in the root: (B) $3 \times 30 \times 2^2 = 360$.

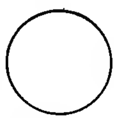
3d—The cube of the last figure obtained: (C) $2^3 = 8$. If the sum of these various products (A) 5400, (B) 360, (C) 8 = 5768, is larger than 6076, it indicates that the trial figure is too large, and a figure one unit smaller should be used.

After having subtracted 5768 from 6076 move down the next period of 3 figures and annex it to the remainder, and proceed as before.

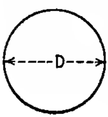
EXERCISES

1. Find the third power of 321.
2. Find the ninth power of 5.
3. Find the fifth power of 11.
4. Find the cube root of 24389.
5. Find the cube root of 997002999.
6. Find the cube root of 99700.2999
7. Find the cube root of 0.4219.
8. Find the cube root of 512000.

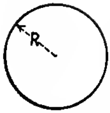
The Circle



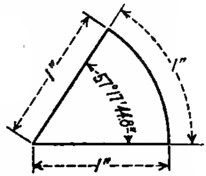
A **Circle** is a plane figure bounded by a line every point of which is an equal distance from the center point.



The **Diameter** of a circle is the distance in a straight line through the center from one edge to the other.

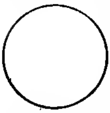


The **Radius** is one-half of the diameter or the distance from the center point of the circle to any point on the circumference measure in a straight line.



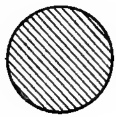
A **Radian** is a part of a circle of such magnitude that the length of the arc is equal to the radius.

The included angle of a radian is equal to 57° 17′ 44.8″.



The **Circumference** of a circle is the distance around and is found by multiplying the diameter by 3.141601 roughly $3\frac{1}{7}$. The figure 3.1416 is sometimes called **Pi** represented by the character π. Example: Find the circumference of a circle 3″ in diameter.

Rule.—Circumference $= D \times \pi = 3'' \times 3.1416$ $= 9.4248''$ (Ans.).



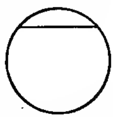
The **Area** of a circle can be found by multiplying the diameter squared by 0.7854 or $(\frac{1}{4}\pi)$.

Rule.—Area $= D^2 \times 0.7854$.

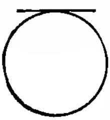
Example: Find the area of a circle 3″ in diameter.

$$A = D^2 \times 0.7854 = A = 3^2 \times 0.7854$$
$$= A = 9 \times 0.7854 = 7.0686 \text{ sq. in. (Ans.).}$$

An **Arc** of a circle is part of its circumference.



A **Chord** is a straight line joining two points on the circumference.



A **Tangent** of a circle is a straight line which touches the circumference at one point only.

A **Sector** of a circle is the space between an arc and two radii drawn to its extremities.

To find the area of a sector of a circle:

Rule I.—Multiply the length of the arc by $\frac{1}{2}$ the radius $= A = L \times \frac{R}{2}$.

Rule II.—$\frac{\text{Angle of sector in degrees}}{360} \times \pi \times$ radius squared $= \frac{\phi}{360}\pi R^2$.

Example I: Find the area of a sector of a circle which has an arc length of 4″ and a 3″ radius.

Rule I.—$A = L \times \frac{R}{2} = 4 \times \frac{3}{2} = 4 \times 1\frac{1}{2} = 6$ sq. in. (Ans.).

Example II: Find the area of a 60° sector which has a radius of 5″.

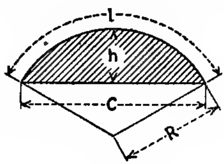
Rule II.—$A = \frac{\phi}{360} \times \pi R^2 = \frac{60}{360} \times 3.1416 \times 5^2$

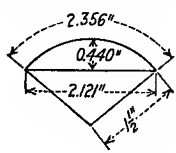
$= \frac{60}{360} \times 3.1416 \times 25 = 13.09$ sq. in. (Ans.).

A **Segment** of a circle is the space between an arc and its chord. To find the area of a segment of a circle:

Rule I.—$A = \frac{1}{2}[R(l - c) + hc]$.

Example I: Find the area of the following segment.





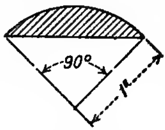
Rule.—$A = \frac{1}{2}[R(l - c) + hc] = \frac{1}{2}[1\frac{1}{2}(2.356 - 2.121)$

$+ .440 \times 2.121] = \frac{1}{2}[1\frac{1}{2}(0.235) + 0.933]$

$= \frac{1}{2}[0.352 + 0.933] = \frac{1}{2}[1.285] = 0.642$ sq. in. (Ans.).

Rule II.—The area of a segment may be found by sub-

tracting from the area of the inclosed sector the area of the inclosed triangle.



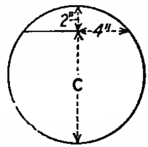
Example II: Find the area of the segment of the following figure.

The area of a 1″ circle = 3.1416 sq. in.

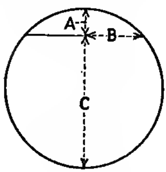
90° = $\frac{1}{4}$ of a circle.

Therefore $\frac{1}{4}$ of 3.1416 sq. in. = 0.7854 sq. in. = area of inclosed sector. The area of the triangle = $\frac{1 \times 1}{2}$ = 0.500 sq. in.

0.7854 sq. in. − 0.500 sq. in. = 0.2854 sq. in. = area of segment.



A line drawn at right angles to a diameter (of any circle) will be a mean proportional to the two parts of the diameter divided, *i.e.*, the length $A : B :: B : C$.



Example: In the following figure A and B given as 2″ and 4″ respectively to find C.

$A : B :: B : C = 2 : 4 :: 4 : 8$. 8 (Ans.).

EXERCISES

1. What area will a sector of a circle whose diameter is $2\frac{1}{4}$″ have if the chord is $1\frac{1}{8}$″ long?

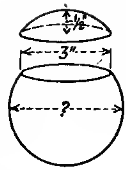
2. Two round shafts of equal diameter will just fit side by side into a $4\frac{1}{2}$″ pipe. What is the radius of the shafts?

3. If a locomotive drive wheel is 6 ft. in diameter, how long will it take to travel $\frac{1}{2}$ mile, running at 200 revolutions per minute?

4. If a lathe job is 4″ in diameter and is turning three revolutions per minute, how far would a point on the circumference travel in a circular direction in one minute's time?

5. If one pint of paint will cover 20 square feet of surface, how much paint would be required to paint a circular clock dial 12 ft. in diameter?

6. Find the number of gallons of water held by a cylindrical tank 4 feet in diameter 3 feet high.



7. If a piece $\frac{1}{2}''$ thick is cut from the face of a sphere and the diameter of the section cut is 3", what is the diameter of the sphere?

8. Find the area of a $15\frac{1}{2}''$ circle,—also circumference.

9. If a ball 10" in diameter be flattened on one side to reduce the diameter to 9", what will be the area of the flat face?

10. If a 4 ft. car wheel has a flat spot on its circumference 6" long how far will the axle drop when this spot reaches the rail?

11. If boiler pressure is 150 lbs., what force will be exerted upon a piston 10" in diameter?

12. Find the area of a sector of a circle, the angle being 42 deg. and the radius 15 inches.

13. Find the area of a sector of a circle which has a 30" arc and a radius of 4 ft.

14. Find the area of a segment which has a 2" radius, 3.1416" arc, 2.828" chord and a rise of 0.586 inches.

15. Find the area of a circular segment which has $\frac{5}{8}''$ radius and the included sector angle is 90°.

16. Find the area of a circular segment which has a 1" radius and the included angle is 60°.

Mensuration and Geometry

A **Point** has position but not magnitude.

A **Line** has length, but neither breadth nor thickness.

A **Surface** has length, breadth but not thickness.

A **Solid** has length, breadth and thickness.

A **Plane** is a surface which is straight in every direction, that is, one which is perfectly flat.

Parallel Lines are straight lines lying in the same plane that are everywhere equally distant from each other. Circular lines which answer to this condition are said to be concentric.

An **Angle** is the difference in the direction of two lines. If the lines meet, that point of meeting is called the **Vertex** of the angle, and the lines are called its sides.

A **Right Angle** is a 90 degree angle—(*a–c–d*). Fig. I.

An **Obtuse Angle** is one which is greater than a right angle—(*a–c–e*).

An **Acute Angle** is one which is less than a right angle—(*e–c–b*).

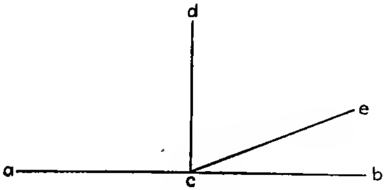


FIG. I

The **Complement of an Angle** is a right angle or 90 degrees, less the given angle. Thus, *b–c–e* is the complement of *d–c–e*.

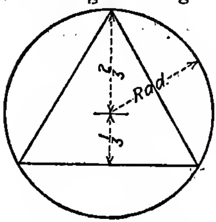
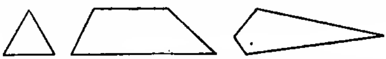


FIG. II

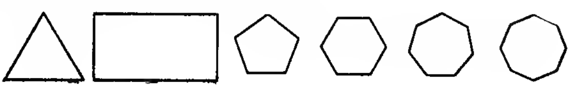
The **Supplement of an Angle** is two right angles or 180 degrees, less the given angle. Thus *b–c–e* is the supplement of *a–c–e*.

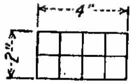
The center of a circle that just includes an equilateral triangle is at a point $\frac{1}{3}$ of the altitude of the triangle from the base (Fig. II).

A **Polygon** is a plain surface bounded by straight lines only. Triangles, rectangles, pentagons, hexagons, heptagons, octa-

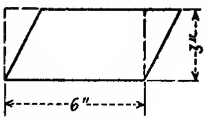


gons, decagons, dodecagons, etc., are all polygons. The areas of irregular shaped surfaces can be found by dividing them into triangles, rectangles, etc.



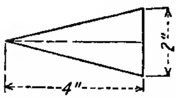


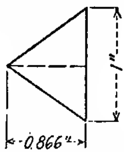
The **Area of a Rectangle** is equal to the length times the breadths (units being similar). Example: Find the area of the following figure: $2 \times 4 = 8$ sq. in. (Ans.).

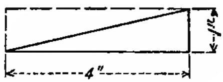


The **Area of a Parallelogram** is equal to the altitude times the base. Example: Find the area of the following figure: $3 \times 6 = 18$ sq. in. (Ans.).

The **Area of an Isosceles, Equilateral, or Right Angle Triangle** is equal to the length times one-half the breadth or one-half of the length times the breadth. Examples: Find the area of the following figures:

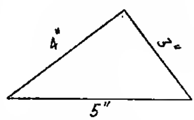




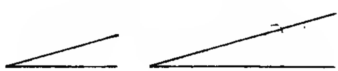


$4 \times 1'' = 4$ sq. in, (Ans.). $0.866'' \times \frac{1}{2}'' = 0.433$ sq. in. (Ans.). $4'' \times \frac{1}{2}'' = 2$ sq. in. (Ans.).

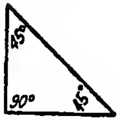
The **Area of Any Shaped Triangle** can be found by subtracting from one-half the sum of the three sides each side severally, then extracting the square root of the product of the three remainders and one-half the sum of the three sides. Example: Find the area of the following figure: $4 + 3 + 5 = 12$ = sum of three sides; $12 \div 2 = 6$ = one-half the sum of the three sides; $6 - 4 = 2$; $6 - 3 = 3$; $6 - 5 = 1$. Then $\sqrt{2 \times 3 \times 1 \times 6} = \sqrt{36} = 6$ (Ans.).



Two angles are equal when their intercepting lins aree parallel or are at right angles. Example:



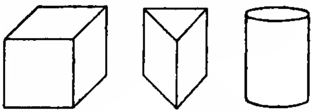
The sum of the three angles in any triangle always equals $180°$. Examples:



$45° + 90° + 45° = 180°$

$40° + 120° + 20° = 180°$

The **Volume of a Cube, Prism or Cylinder** is equal to the area of the base times the height.



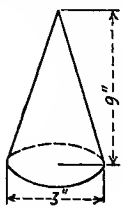
The **Volume of a Sphere** $= \frac{1}{6}D^3 \times \pi$. Example: Find the volume of a 3″ sphere.

Rule.—$V = \frac{1}{6}D^3 \times \pi = \frac{1}{6}3^3 \times 3.1416 = \frac{1}{6}27 \times 3.1416$
$= \frac{1}{6}84.81 = 14.13$ cu. in. (Ans.).

The **Convexed Area of a Sphere** $= D^2 \times \pi$. Example: Find the surface area of a sphere 3″ in diameter.

Rule.—$A = D^2 \times \pi = 3^2 \times 3.1416 = 9 \times 3.1416$
$= 28.27$ sq. in. (Ans.).

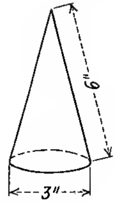
The **Volume of a Cone** = area of base $\times \frac{1}{3}$ of the altitude. Example: Find the volume of the following figure.



Rule.—$V = A \times \dfrac{H}{3}$.

A = area.
H = height or altitude.

$V = A \times \dfrac{H}{3} = 7.07 \times \dfrac{9}{3} = 7.07 \times 3 = 21.21$ cu. in. (Ans.).



To find the **Total Surface Area of a Cone:**

Rule.—Surface Area $= C \times \frac{H}{2} + A.$

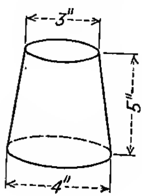
C = circumference of base.
H = slant height.
A = area of base.

Example: Find the total surface area of the above cone:

Rule.—Surface Area $= C \times \frac{H}{2} + A = 3 \times 3.1416 \times \frac{6}{2}$

$+ 3^2 \times 0.7854 = 9.42 \times 3 + 7.07 = 28.26 + 7.07$
$= 35.33$ sq. in. (Ans.).

To find the **Volume of a Frustum of a Cone:**

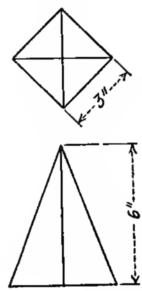


Rule.—$V = 0.2618H(D^2 + Dd + d^2)$.

Example: Find the volume of the following frustum:

Rule.—$V = 0.2618H(D^2 + Dd + d^2)$
$= 0.2618 \times 5(4^2 + 4 \times 3 + 3^2)$
$= 1.309(16 + 12 + 9)$
$= 1.309 \times 37 = 48.436$ cu. in. (Ans.).

To find the **Volume of a Pyramid:**



Rule.—$V = \frac{H}{3} \times A.$

H = height.
A = area of base.

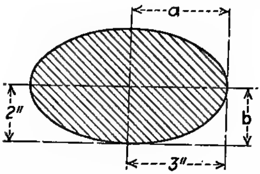
Example: Find the volume of the following pyramid:

Rule.—$V = \frac{H}{3} \times A = \frac{6}{3} \times 9 = 2 \times 9 = 18$ cu. in. (Ans.).

To find the **Area of an Ellipse:**

Rule.—$A = \pi \times a \times b.$

Example: Find the area of the following ellipse:



Rule.—$A = \pi \times a \times b$

$= 3.1416 \times 3 \times 2$

$= 18.85$ sq. in. (Ans.).

To find the **Circumference of an Ellipse:**

Rule.—**Cir.** $= \pi \sqrt{2(a + b^2)}$.

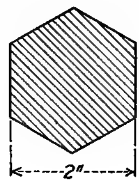
Example: Find the circumference of the above ellipse:

Rule.—**Cir.** $= \pi \sqrt{2(a^2 + b^2)} = 3.1416 \sqrt{2(3^2 + 2^2)}$

$= 3.1416 \sqrt{2(9 + 4)} = 3.1416 \sqrt{2 \times 13} = 3.1416 \sqrt{26}$

$= 3.1416 \times 5.099 = 16.019$ in. (Ans.).

To find the **Area of a Hexagon:**



Rule.—$A = D^2 \times 0.866$.

Example: Find the area of the following hexagon:

Rule.—$A = D^2 \times 0.866 = 2^2 \times 0.866$

$= 4 \times 0.866 = 3.464$ sq. in. (Ans.).

The volume of any irregular shaped object may be found by the displacement method. For example: Place an object in a vessel 10″ long and 6″ wide and pour in sufficient water to completely submerge the object. For convenience let us imagine that the water is $7\frac{1}{2}$″ deep; then by taking the object out of the water, the water will lower in level an amount equal to the volume of the object. Thus if the water was $7\frac{1}{2}$″ deep at first and after the object is taken out it is measured and found to be $6\frac{1}{4}$″ deep or a decrease of $1\frac{1}{4}$″. Thus the total displacement $= 6 \times 10 \times 1\frac{1}{4} = 75$ cu. in. or the volume of the object submerged in cu. in.

The following table gives the ratio of the length of side to the diameter of various polygons.

Example: What must the points of a pair of dividers be set to, to space off a 2″ circle into 5 equal parts?

Opposite figure 5 is found the figure 0.588 or the distance

for one inch; thus for 2″ the dividers would be set to 2 × 0.588″ or 1.176″ (Ans.).

Example: What is the length of one side of an octagon inscribed in a 3″ circle?

Opposite division No. 8 is found the figure 0.383 or the length of side for a one inch circle divided into 8 parts. Thus for a 3″ circle the length of side is equal to 3 × 0.383″ or 1.149″ (Ans.).

No. of Divisions	Degrees of Arc	Length of Side when Diameter = 1	No. of Divisions	Degrees of Arc	Length of Side when Diameter = 1
3	120′	0.866	12	30	0.259
4	90	0.707	13	27° 41′	0.239
5	72	0.588	14	25° 42′	0.222
6	60	0.500	15	24	0.208
7	51° 25′	0.434	16	22° 30′	0.195
8	45	0.383	17	21° 11′	0.184
9	40	0.342	18	20	0.174
10	36	0.309	19	18° 57′	0.164
11	32° 43′	0.282	20	18	0.156

EXERCISES

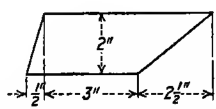
A. Draw free hand, a surface.

B. Draw free hand, a polygon.

C. Draw free hand, a hexagon.

D. Draw free hand, an octagon.

1. Find the area of the following figure:

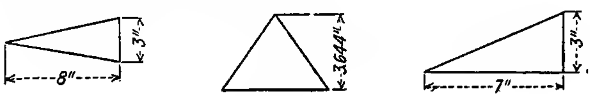


2. Find the area in sq. ft. of a lot 40′ × 120′.

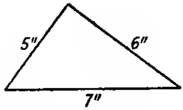
3. Find cost at 12 cents per sq. ft. for cementing a garage floor 20′ × 20′.

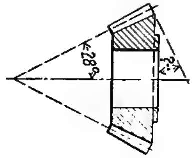
4. What will it cost to shingle a church steeple which is 12 ft. square at the base and has a slant height (measured on the corner) of 30 ft., providing one bundle of shingles cost 75 cents, and will cover 25 sq. ft., and full bundles must be paid for?

5. Find the area of the following:

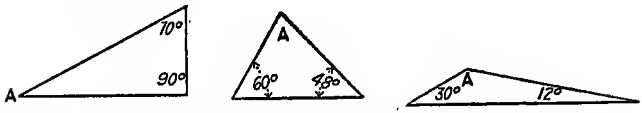


6. Find the area of the following:





7. Find angles A:



8. If the center angle on a bevel gear is 28 deg., what angle would the back end of the tooth make with the end of the hub?

9. What is the capacity in gallons of a cylindrical tank 3′ in diam. and 8′ high?

10. How many cubes of iron 2″ square will go into a box $3' \times 2' \times 1\frac{1}{4}'$?

11. Find the volume of a triangular prism 1″ face, 3″ high.

12. Find the convex area and volume of a sphere 3″ in diameter.

13. Find the total surface area and volume of a cone 12″ in diameter and 15″ slant height.

14. How many cubic yards of earth will be removed from a round well 4 ft. in diameter and 40 ft. deep?

15. If cast iron weighs 0.26 lbs. per cubic inch, what will a 3″ diameter round bar weigh 24″ long?

16. What will be the cost of plating both sides of a sheet of iron 24″ in diameter at 30 cents per sq. ft.?

17. Find the area and circumference of an ellipse with major axis of 18″ and minor axis of 8″.

18. Find the volume of a frustrum of a cone 6″ high, small diameter 2″ and large diameter 4″.

19. What is the volume in cubic inches of a casting which when submerged in a vessel of liquid 6″ in diameter, raises the level of the liquid $2\frac{1}{8}$″?

20. In a vessel 6″ wide and 8″ long are placed 5 castings. What is the volume of each casting in cu. in. if the level of the liquid was raised from $3\frac{1}{4}$″ deep to $6\frac{3}{4}$″?

21. What should a pair of dividers be set to, if it is desired to space a circle 3″ in diameter into 7 equal parts?

22. What is the length of one side of a pentagon which is inscribed in a 4″ circle?

23. What is the perifery length of an octagon inscribed in a 3″ circle?

24. What should the dividers be set to, in spacing off an 8″ circle into 15 equal parts on the circumference?

REVIEW EXERCISES

1. How many gallons of water will a tank hold which is 10 ft. long, 6 ft. wide and 4 ft. deep? (231 cu. in. = 1 gal.)

2. How many military paces will be required to travel 6 furlongs?

3. How many feet are there in half a league?

4. At 20 cents per yard for chain, what will be the cost of 40 fathoms?

5. If stone weighs 175 lbs. per cubic foot, what will 5 perch weigh?

6. How many bushels of wheat can be placed in a bin 6 ft. deep 8 ft. wide and 10 ft. long?

7. What horsepower is lost per hour, if 40720 B.t.u. are lost through the exhaust steam of a steam engine in a 10 hr. run?

8. How many feet of lumber are there in 10 pieces of 2″ × 4″, 16 ft. long?

9. If a gas engine has a bore of 90 millimeters, what is its bore in inches?

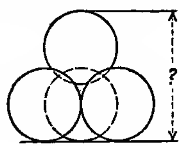
10. What will a tank of water weigh which is 3 ft. deep, 4 ft. wide and 6 ft. long?

11. What will be the weight in grains of a $1\frac{5}{8}$ carat diamond?

12. What horsepower is represented by 3,000 k.w. generator?

13. What will a cast iron block 3″ × 4″ × 8″ with a 1″ square hole cored lengthwise weigh?

14. If 1 pound of carbon burned to CO_2 produces 14,600 B.t.u., how much energy in foot pounds should be developed by burning 10 pounds of carbon?



15. What is the difference in weight between bars of aluminum and steel 1″ × 2″ × 12″?

16. What will be the height over all of a pyramid composed of 4 steel balls 1″ in diameter?

PART II

Formulas and Algebraical Expressions

A **Formula** is a rule for a calculation expressed by using letters and signs instead of writing out the rule in words. The letters used (algebraical expressions) simply stand in place of the figures which are to be substituted when solving problems.

Any formula may be transposed, that is, the value of any letter in the formula may be found in terms of the other letters by following certain set rules. Example: $b = a \times c$ then $a = b/c$, $c = b/a$.

Any quantity may be moved from one side of the equality sign to the other by changing its sign, thus $a - b = c$ then $a = c + b$.

When several numbers or quantities in a formula are connected with signs indicating that additions, subtractions, multiplications or divisions are to be performed, the multiplications should be carried out before any of the other operations, and division also preceeds additions or subtractions, as $18 \div 6 + 15 \times 3 - 2 = 3 + 45 - 2 = 46$.

When it is desired that certain additions and subtractions should preceed multiplications or divisions, use is made of vinculum $\overline{\qquad}$, parentheses ( ), brace { }, and brackets [ ]. These indicate that the calculations included in the $\overline{\qquad}$ ( ) { } [ ] shall be carried out complete by itself, before the remaining calculations are commenced. If a $\overline{\qquad}$ ( ) { } or [ ] is placed inside of one another, the one inside is first calculated. The order of use is as follows $[\{(\overline{\qquad})\}]$. Exmple: $2 + [10 \times 6(8 + \overline{4 - 2}) - 4] \times 2 = 2 + [10 \times 6 \times 10 - 4] \times 2 = 2 + [600 - 4] \times 2 = 2 + 596 \times 2 = 2 + 1192 = 1194$ (Ans.).

The line or bar between the numerator and the denominator of any fraction is to be considered as a division sign, thus:

$$\frac{12 + 16 + 22}{10} = \frac{50}{10} = 50 \div 10 = 5 \text{ (Ans.).}$$

In formulas the multiplication sign ($\times$) is often omitted between symbols or letters, the values of which are to be multiplied thus $a \times b = ab$ and $abc/d = (a \times b \times c) \div d$.

A $\overline{\quad\quad}$ ( ) { } or [ ] may be removed by performing the operations indicated thus, $a(b + c) = ab + ac$.

When a $\overline{\quad\quad}$ ( ) { } or [ ] preceded by a minus sign ($-$) is removed, all signs within must be changed, thus $a - b(c - d) = a - bc + bd$.

A period is sometimes used as a multiplication sign, thus $a.b = a \times b$.

A radical sign $\sqrt{\ }$ indicates that a root is to be found. The small figure $\sqrt[3]{\ }$ above the radical is called the index of the root. A radical is removed by extracting the root or raising both terms to the power of the index, thus, if $\sqrt[3]{a} = b$ then $a = b \times b \times b = b^3$.

In the **Addition** of algebraical quantities, all like quantities are placed in the same column, thus: $7a + 2b + 4c + 3b + 5a + 3c =$

$$\begin{array}{r} 7a + 2b + 4c \\ 5a + 3b + 3c \\ \hline 12a + 5b + 7c \end{array} \text{ (Ans.).}$$

When like quantities have the same sign, their sums are found by simply adding the numbers and annexing the common letter, thus: $5a + 2b + 7a + 3b =$

$$\begin{array}{r} 5a + 2b \\ 7a + 3b \\ \hline 12a + 5b \end{array} \text{ (Ans.).}$$

When like quantities do not have the same sign, the

positive (+) and negative (−) signs must be added separately, and afterward the smaller group subtracted from the greater and the sign of the greater is prefixed and the common letter or letters added, thus: $7a - 2b - 4c - 5a + 3b + 3c =$

$$\begin{array}{r} 7a - 2b - 4c \\ -\ \underline{5a + 3b + 3c} \\ 2a + \ b - \ c \end{array} \text{ (Ans.).}$$

In the **Subtraction** of algebraical quantities, the signs of all terms subtracted must be changed, and then all like terms are added as above mentioned, thus: Subtract $5x^2 - 2x + 6$ from $10x^2 + 4x - 2 =$

$$\begin{array}{r} 10x^2 + 4x - 2 \\ \underline{5x^2 - 2x + 6} \end{array} = \begin{array}{r} 10x^2 + 4x - 2 \\ -\ \underline{5x^2 + 2x - 6} \\ 5x^2 + 6x - 8 \end{array} \text{ (Ans.).}$$

In the **Multiplication** of algebraical quantities, the rules of signs must be observed, *i.e.*, the product of two terms of similar signs is positive (+), thus: $2 \times +3 = 6$, or $-2 \times -3 = +6$.

The product of two dissimilar terms is negative (−), thus: $2 \times -3 = -6$, or $-2 \times +3 = -6$.

When multiplying two simple expressions together, first multiply the figures and then add the exponents of the letters, thus: $4ab \times 2a^2b^2 =$

$$\begin{array}{r} 4a\ b \\ \times\ \underline{2a^2b^2} \\ 8a^3b^3 \end{array} \text{ (Ans.).}$$

Or multiplying $a + 4$ by $a + 5 =$

$$\begin{array}{l} a + 4 \\ \underline{a + 5} \\ a^2 + 4a \\ \underline{\qquad\ \ 5a + 20} \\ a^2 + 9a + 20 \end{array} \text{ (Ans.).}$$

Or multiplying $2a + 3b - 4x$ by $2a - 3b - 4x =$

$$\begin{array}{lllll}
2a + 3b & - 4x & & & \\
2a - 3b & - 4x & & & \\
\hline
4a^2 + 6ab & - 8ax & & & \\
\quad - 6ab & & - 9b^2 & + 12xb & \\
 & - 8ax & & - 12xb & + 16x^2 \\
\hline
4a^2 & - 16ax & - 9b^2 & & + 16x^2 \text{ (Ans.)}
\end{array}$$

In the **Division** of algebraical quantities, the rules of signs must also be observed, *i.e.*, the sign of any term of the quotient is positive (+) when the dividend and the divisor have like signs and negative (−) when they have unlike signs.

In division the coefficient of the quotient is equal to the coefficient of the dividend divided by the divisor, thus: $20ab \div 5ab = 4$ (Ans.) or $10a \div 2a = 5$ (Ans.) or $30a^2b^3 \div - 6a^2b = - b^2$ (Ans.).

In the division of algebraical quantities, cancellation can be used to simplify the operation. Thus:

$$\frac{24a^3b^2x}{10a^2b\,x^2} = \frac{12ab}{5x}.$$

The use of formulas can possibly be best explained by a few simple problems as follows:

(1) Find the area of a circle 2″ in diameter?

The formula for finding the area of a circle $= D^2 \times 0.7854$.

D = diameter of circle.

0.7854 = constant.

$$D^2 \times 0.7854 = 2^2 \times 0.7854 = 4 \times 0.7854$$
$$= 3.1416 \text{ sq. in. (Ans.)}.$$

(2) What is the S.A.E. rating of a gas engine, that has 8 cylinders with a 3″ bore, running at 2000 f.p.m.?

The S.A.E. formula $= \dfrac{D^2 \times N}{2.5}$ for a piston speed of 1000 f.p.m.

D = diameter of cylinder in inches.
N = number of cylinders.
2.5 = constant.

$$\frac{D^2 \times N}{2.5} = \frac{3^2 \times 8}{2.5} = \frac{9 \times 8}{2.5} = 28.8.$$

$$28.8 \times 2 = 57.6 \text{ h.p. (Ans.).}$$

(3) What is the i.h.p. of a steam engine running at 125 r.p.m., with a 10″ bore and a 12″ stroke, with a m.e.p. of 50 lbs. per sq. inch?

The formula for i.h.p. $= \dfrac{2\ PLAN}{33000}$.

P = mean effective pressure in lbs. per sq. inch.
L = length of stroke in feet.
A = area of piston in sq. inches.
N = number of revolutions per minute.
2 = a constant which is necessary because a steam engine is a double acting engine, and therefore obtains 2 power impulses per revolution.

33000 = number of ft. lbs. per minute, equivalent to 1 h.p.

$$\frac{2\ PLAN}{33000} = \frac{2 \times 50 \times 78.54 \times 125}{33000} = 29.75 \text{ i.h.p. (Ans.).}$$

(4) What is the centrifugal force produced by a 10 lb. weight fastened to the end of a cord 5 ft. long, revolving at a velocity of 20 ft. per second?

Formula $F = \dfrac{WV^2}{gR}$.

F = centrifugal force.
W = weight in lbs.

V = velocity in ft. per second.
g = gravity (32.16).
R = radius.

$$F = \frac{WV^2}{gR} = \frac{10 \times 20^2}{32.16 \times 5} = \frac{10 \times 400}{32.16 \times 5} = 24.87 \text{ lbs. (Ans.).}$$

(5) What h.p. will a 2″ rope transmit, running at 5000 f.p.m.?

Formula = h.p. = $D^2 \times V \times 0.003 \times N$.
D = diameter of rope in inches.
V = velocity in f.p.m.
0.003 = constant.
N = number of ropes.

$$\text{h.p.} = D^2 \times V \times 0.003 \times N = 2^2 \times 5000 \times 0.003 \times 1$$
$$= 4 \times 5000 \times 0.003 \times 1 = 60 \text{ h.p. (Ans.).}$$

If h.p. = $D^2 \times V \times 0.003 \times N$, then the formula to find the diameter of the rope required can be obtained by substitution, which will become—

$$D = \sqrt{\frac{\text{h.p.}}{V \times 0.003 \times N}} = D = \sqrt{\frac{60}{5000 \times 0.003 \times 1}}$$
$$= D = \sqrt{4} = 2'' \text{ (Ans.).}$$

To find the number of ropes required, the formula by substitution will be—

$$N = \frac{\text{h.p.}}{D^2 \times V \times 0.003} = N = \frac{60}{4 \times 5000 \times 0.003} = 1 \text{ (Ans.).}$$

EXERCISES

1. (a) Add $6a + 5b - 3a - 4b + 6a - 3b + x$.
   (b) Add $a + 2b + c + a - b + a + 3b + 2c$.
2. (a) Subtract $3a^2 - 2b + 8c$ from $5a^2 - 4b + 6c$.
   (b) Subtract $12xy^2z^3$ from $8xy^2z^3$.
3. (a) Multiply $a + 6$ by $a + 4$.
   (b) Multiply $x + y$ by $x - y$.

4. (*a*) Divide $12ab^2$ by $6ab$.
(*b*) Divide $-20x^3y^2$ by $-5x^2y$.

5. Find the value of $\dfrac{ab^2 + y^2}{yb - a^2 - c}$ when $a = 3$, $b = 6$, $c = 2$, $y = 5$.

6. Find the value of $\dfrac{3x + 4y}{3z - x - y} - \dfrac{x^2 - 2y^2}{z^2 - y^2}$ when $x = 10$, $y = 5$, $z = 7$.

7. Find the value of $\sqrt{\dfrac{3b + x}{x - b}} - \sqrt{\dfrac{9b + 7x - 12}{x - b}}$ when $a = 10$, $b = 3$, $x = 7$.

8. What is the true length of one coil of a spring which is 2″ in diameter and a $\frac{1}{4}$″ lead?

Formula for length of one coil $= \sqrt{(2\pi r)^2 + L^2}$.

r = radius of mean diameter in inches.
L = lead.

9. What is the safe load in tons of a 6″ × 6″ sq. cast iron column 10 ft. long?

The formula for cast iron columns in tons $= \dfrac{5a}{1 + \dfrac{L^2}{1000d^2}}$.

a = sectional area in sq. inches.
L = length of column in inches.
d = width of sides of column in inches.

10. What is the b.h.p. developed by a steam engine with a Prony brake, if the length of brake arm is 6 ft., the weight registered on the scale is 100 lbs. and the engine is running 120 r.p.m.?

Formula for Prony brake h.p. $= \dfrac{2\pi LWN}{33000}$.

L = length of brake arm in ft.
W = weight or pressure at end of arm in lbs.
N = number of revolutions per minute.
33000 = constant.

11. The formula for the horsepower transmitted by belting is as follows:

$$\text{h.p.} = \frac{SVW}{33000},$$

in which h.p. = horsepower transmitted,

S = working stress of belt per inch of width in lbs.,
V = velocity of belt in ft. per min.,
W = width of belt in inches,

find the values of S, V and W.

12. The formula for the horsepower of a steam engine is

$$\text{i.h.p.} = \frac{2PLAN}{33000},$$

in which i.h.p. = indicated horsepower,

P = mean effective pressure in lbs. per sq. in.,
L = length of stroke in feet,
A = area of piston in sq. in.,
N = number of revolutions per min.,

find the values of P, L, A and N.

13. In the formula $A = \pi(ab - cd)$ find the values of a, b, c and d.

14. Simplify

$$0 = 2\left[U - U\left\{X + Z\left(T - M + \frac{N}{P}\right)\right\}\right].$$

15. In the formula

$$W = \frac{A(V + S)}{P}$$

find the values of A, V, S and P.

16. In the formula for the space passed through by a falling body, starting from rest, $S = \frac{1}{2}gt^2$, in which

S = space body falls in feet,
g = acceleration of gravity in ft. per sec. per sec.,
t = time of fall in seconds,

find the value of t and g.

17. In the formula for the space passed through by a falling body which starts with an initial velocity

$$S = \frac{V^2 - v^2}{2g}.$$

in which S = space body falls in feet,
g = acceleration of gravity,
V = final velocity in ft. per sec.,
v = initial velocity in ft. per sec.,

find the values of V, v and g.

18. In the formula for triangles $H^2 = A^2 + B^2$, in which

H = hypotenuse,
A = altitude,
B = base,

find H, A and B.

19. In the formula

$$Z = \frac{M^2 \times N^2}{S}$$

find the values of M, N and S.

20. In the formula

$$D = \frac{5H}{N + M}$$

find the values of H, N and M.

Progression

Progression means a progressive increase or decrease of a series of numbers.

There are two kinds of progression in general use—arithmetical and geometrical.

Arithmetical Progression in a series of numbers is a progressive increase or decrease in each term by adding or subtracting a constant amount called the **Common Difference.**

Thus the figures 1–3–5–7–9–11–13 etc., is an increase progression with a common difference of 2.

While 15–12–9–6–3–0 is a decreasing progression with a common difference of 3.

In arithmetical progression the following terms and formulas are generally used.

A = first term,
L = last term,
N = number of terms,
D = common difference,
S = sum of all terms.

Formulas

$$L = A + (N - 1)D, \qquad D = \frac{L - A}{N - 1},$$
$$A = L - (N - 1)D,$$
$$N = 1 + \frac{L - A}{D}, \qquad S = \frac{A + L}{2D} \times (L + D - A).$$

Example: Find the last term in an arithmetical progression of 5 figures, if the first term is 2 and the difference is 3.

Formula: $L = A + (N - 1)D = 2 + (5 - 1)3$
$= 2 + 4 \times 3 = 2 + 12 = 14$ (Ans.).

Example: Find the first term in an arithmetical progression of a series of 5 numbers, if the difference is 2 and the last number is 11.

Formula: $A = L - (N - 1)D = 11 - (5 - 1)2$
$= 11 - 4 \times 2 = 11 - 8 = 3$ (Ans.).

Example: Find the number of terms in an arithmetical progression, if the first term is 2, the last is 20 and the difference is 3.

Formula: $N = 1 + \frac{L - A}{D} = 1 + \frac{20 - 2}{3} = 1 + \frac{18}{3}$
$= 1 + 6 = 7$ (Ans.).

Example: Find the common difference in the following arithmetical progression, first number is 3, the last 21 in a series of 6 numbers.

Formula: $D = \frac{L - A}{N - 1} = \frac{21 - 3}{7 - 1} = \frac{18}{6} = 3$ (Ans.).

Example: Find the sum of the arithmetical progression, if the first term is 3 and the last is 28, if the difference is 5.

Formula: $S = \frac{A + L}{2D} \times (L + D - A) = \frac{3 + 28}{2 \times 5}$
$\times (28 + 5 - 3) = \frac{31}{10} \times 30 = 3.1 \times 30 = 93$ (Ans.).

Geometrical Progression in a series of numbers is a progressive increase or decrease in each term obtained by multiplying or dividing the preceding term by a constant called the **Ratio.**

Thus 1–2–4–8–16–32 etc., is an increasing geometrical progression with a ratio of 2 and 81–27–9–3–1 is a decreasing geometrical progression with a ratio of 3.

In geometrical progression the following terms and formulas are generally used.

A = first term,
L = last term,
N = number of terms,
R = ratio,
S = sum of all terms.

Formulas

$$A = \frac{L}{R^{N-1}}, \qquad L = AR^{N-1}.$$

$$N = \frac{\log L - \log A}{\log R} + 1, \qquad R = \frac{S-A}{S-L}.$$

$$S = \frac{LR - A}{R - 1},$$

Example: Find the first number of a geometrical progression of five terms, if the last number is 243 and the ratio is 3.

Formula: $A = \frac{L}{R^{N-1}} = \frac{243}{3^{5-1}} = \frac{243}{3^4} = \frac{243}{81} = 3$ (Ans.).

Example: Find the last term in a geometrical progression of 5 figures, if the first term is 1 and the ratio is 2.

Formula: $L = AR^{N-1} = 1 \times 2^{5-1} = 1 \times 2^4$
$= 1 \times 16 = 16$ (Ans.).

Example: Find the number of terms in a geometrical progression, if the first term is 2, the last is 486, if the ratio is 3.

Formula: $N = \frac{\log L - \log A}{\log R} + 1 = \frac{\log 243 - \log 3}{\log 3} + 1$

$$= \frac{2.385 - 0.477}{0.477} + 1 = \frac{1.908}{0.477} + 1 = 4 + 1 = 5 \text{ (Ans.)}.$$

Example: Find the ratio in the following geometrical progression: 1–1.3–1.69–2.19–2.85.

Formula: $R = \dfrac{S - A}{S - L}$

$$= \frac{1 + 1.3 + 1.69 + 2.19 + 2.85 - 1}{1 + 1.3 + 1.69 + 2.19 + 2.85 - 2.85} = \frac{8.03}{6.18} = 1.3 \text{ (Ans.)}.$$

Example: Find the sum of a geometrical progression, if the first term is 2, the last $10\frac{1}{8}$, with a ratio of $1\frac{1}{2}$.

Formula: $S = \dfrac{LR - A}{R - 1} = \dfrac{10\frac{1}{8} \times 1\frac{1}{2} - 2}{1\frac{1}{2} - 1}$

$$= \frac{15\frac{3}{16} - 2}{\frac{1}{2}} = \frac{13\frac{3}{16}}{\frac{1}{2}} = 26\tfrac{3}{8} \text{ (Ans.)}.$$

EXERCISES

1. Find the first term in an arithmetical progression in a series of 8 numbers if the difference is 3 and the last number is 25.

2. Find the last term in an arithmetical progression of 6 figures, if the first term is 3 and the difference is 2.

3. Find the sum of the arithmetical progression, if the first term is 4, the last 16 and the difference is 3.

4. Find the number of terms in an arithmetical progression, if the first term is 6, the last 27 and the difference is 3.

5. What is the common difference used in a six step cone pulley, if the small pulley measures 3″ in diameter and the largest one 13″?

6. What is the last term in a geometrical progression of six numbers, if the first term is 1 and the ratio is 3?

7. Find the number of terms in a geometrical progression, if the first term is 4, the last 256 and the ratio is 4.

8. Find the first number of a geometrical progression of 5 figures if the last number is $78\frac{1}{8}$ and the ratio is 2.5.

9. What is the sum of a geometrical progression, if the first term is 8, the last 2.621 and the ratio 0.8?

10. Find the ratio used in a five step cone pulley having the following diameters: 5″–6″–7″–8″–9″.

Trigonometry

Trigonometry is that branch of mathematics which deals with the determination of angles and the solution of triangles.

Any figure bound by three straight sides is called a **Triangle.** The sides are called **Base** (B), **Altitude** (A) and **Hypotenuse** (H) (Fig. I–a), or more conveniently, **Side Adjacent** (SA), **Side Opposite** (SO) and **Hypotenuse** (hyp) (Fig. 1–b). All triangles have three angles, and their sum is always equal to 180 degrees (Fig. I–c).

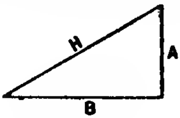


FIG. I-*a*

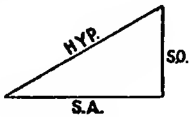


FIG. I-*b*

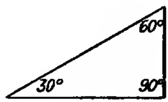
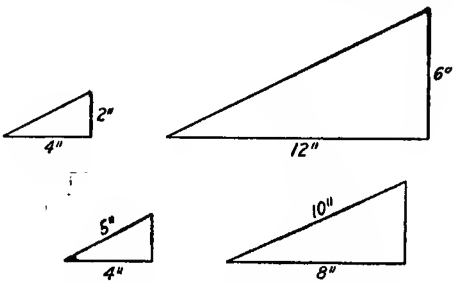


FIG. I-*c*

Two angles are equal when they contain the same number of degrees. Example:

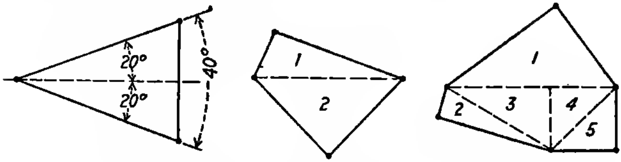
(An angle may be defined as the path through which a ray passes revolving about a fixed point called the center.)

A **Right Triangle** is one which contains a right angle or a 90 degree angle. In all right triangles having the same acute angle, the sides are proportional. Example:



This fact forms the working basis of trigonometry. In

practical work the right triangle is most often encountered, otherwise the figure can easily be subdivided into two or more right triangles, as shown in the following:



There are six possible ratios between the three sides of a right triangle called the trigonometric functions, and designated sine (sin), cosine (cos) tangent (tan), cotangent (cot), secant (sec), and cosecant (csc). These are defined as follows:

(Let X equal the angle in all cases)

$$(1)\quad \text{sine } X = \frac{\text{side opposite}}{\text{hypotenuse}} \quad \text{or} \quad \sin = \frac{SO}{hyp};$$

$$(2)\quad \text{cosine } X = \frac{\text{side adjacent}}{\text{hypotenuse}} \quad \text{or} \quad \cos = \frac{SA}{hyp},$$

$$(3)\quad \text{tangent } X = \frac{\text{side opposite}}{\text{side adjacent}} \quad \text{or} \quad \tan = \frac{SO}{SA},$$

$$(4)\quad \text{cotangent } X = \frac{\text{side adjacent}}{\text{side opposite}} \quad \text{or} \quad \cot = \frac{SA}{SO},$$

$$(5)\quad \text{secant } X = \frac{\text{hypotenuse}}{\text{side adjacent}} \quad \text{or} \quad \sec = \frac{hyp}{SA},$$

$$(6)\quad \text{cosecant } X = \frac{\text{hypotenuse}}{\text{side opposite}} \quad \text{or} \quad \csc = \frac{hyp}{SO}.$$

It will be seen that the last three functions are the reciprocals or inverse of the first three, and this is the easiest way to remember them.

Note.—The foregoing definitions must be mastered before any calculations can be made.

The prefix "co" in the functions comes from "complementary," the functions of an angle being the co-function of a complementary angle.

For example, sine $X = \cos (90 - X)$, $\tan 30 = \cot 60$, $\sec 15 = \csc 75$, etc.

The trigonometric functions are numerical values which can be determined for every angle. However, it would be entirely impractical to determine these values for each application, so complete tables have been compiled giving the values for all angles, and the quantities are taken direct from the tables, sines and tangents are the two most important functions.

Since most calculations can be made with sines, cosines, tangents and cotangents, only these are tabulated in this book.

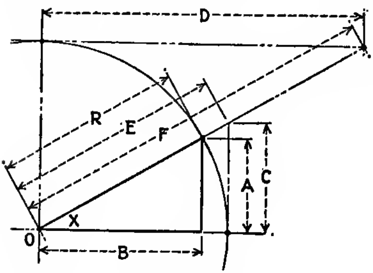


FIG. II

In Fig. II, the functions are illustrated graphically

R = radius = 1,
A = sine,
B = cosine,
C = tangent,
D = cotangent,
E = secant,
F = cosecant.

Since the radius equals 1, the sides or numerators of the ratios are disposed so that the denominator is always the radius, thus permitting the denominator to be disregarded and only the length of the numerator considered.

Note.—(In Fig. II the angle used is 30 degrees. Compare the relative length of the lines with the values given in the tables.)

Thus in Fig. II, if the radius is equal to 1, and the angle is 30 degrees, according to the table of trigonometrical functions:

$$A = .50000,$$
$$B = .86603,$$
$$C = .57735,$$
$$D = 1.7320,$$
$$E = 1.1547,$$
$$F = 2.0000.$$

Sines

The **Sine** of an angle is the relation of the side opposite of any given angle to its hypotenuse, or vise versa.

Rules.—1. Sine $A = \dfrac{\text{side opposite}}{\text{hypotenuse}}$.

2. Side opposite = sine $A \times$ hypotenuse.

3. Hypotenuse $= \dfrac{\text{side opposite}}{\text{sine } A}$.

Problem No. 1: What is the angle in Fig. III, if the side opposite = 0.68404″ and the hypotenuse = 2.000″?

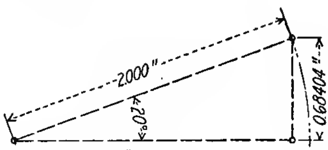


Fig. III

Rule No. 1: Sine $A = \dfrac{\text{side opposite}}{\text{hypotenuse}} = \dfrac{0.68404}{2} = 0.34202.$

Upon looking in the table of sines, we find that sine .34202 is equivalent to 20 degrees (Ans.).

Problem No. 2: What is the length of the side opposite in Fig. III if the angle is 20 degrees and the hypotenuse is 2.000″?

Rule No. 2: Side opposite = sine × hypotenuse.
Since of 20 deg. = .34202.
.34202 × 2 = 0.68404″ = side opposite (Ans.).

Problem No. 3: What is the length of the hypotenuse in Fig. III if the side opposite is 0.68404″ and the angle is 20 degrees?

Rule No. 3: Hypotenuse $= \dfrac{\text{side opposite}}{\text{sine}}$.

Sine of 20 deg. = .34202.

0.68404″ ÷ .34202 = 2.000″ = hypotenuse (Ans.).

(Note that the values of the sines of all angles range from zero (0) to one (1) inclusive.)

Cosines

The **Cosine** of an angle is the relation of the side adjacent to that of the hypotenuse, or vise versa.

Rules.—1. Cosine $= \dfrac{\text{side adjacent}}{\text{hypotenuse}}$.

2. Side adjacent = cosine × hypotenuse.

3. Hypotenuse $= \dfrac{\text{side adjacent}}{\text{cosine}}$.

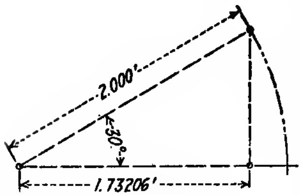


FIG. IV

Problem No. 1: What will the angle be in Fig. IV if the hypotenuse is 2 ft. and the side adjacent = 1.73206 ft.?

Rule No. 1: Cosine $= \dfrac{\text{side adjacent}}{\text{hypotenuse}}$.

1.73206 ÷ 2 = .86603.

Cosine .86603 is equivalent to 30 degrees (Ans.).

Problem No. 2: What is the length of the side adjacent in Fig. IV if the angle is 30 deg. and the hypotenuse is equal to 2 ft.?

Rule No. 2: Side adjacent = cosine × hypotenuse.

Cosine of 30 deg. = .86603. Hypotenuse = 2 ft.

∴ .86603 × 2 = 1.73206 ft. (Ans.).

Problem No. 3: What is the length of the hypotenuse in Fig. IV if the angle is 30 deg. and the side adjacent is equal to 1.73206 ft.?

Rule No. 3: Hypotenuse $= \frac{\text{side adjacent}}{\text{cosine}}$.

Side adjacent = 1.73206 ft. Cosine of 30 deg. = .86603.
$\therefore$ 1.73206 ÷ .86603 = 2 ft. (Ans.).
(Note that the values of the cosines range from zero (0) to one (1) inclusive.)

Tangents

The **Tangent** of any angle is the relation of the side opposite to that of the side adjacent, or vice versa.

Rules.—1. Tangent $= \frac{\text{side opposite}}{\text{side adjacent}}$.

2. Side opposite = side adjacent × tangent.

3. Side adjacent $= \frac{\text{side opposite}}{\text{tangent}}$.

Problem No. 1: What is the angle in Fig. V if the side

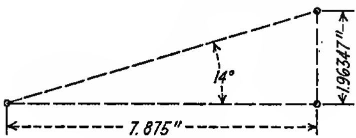


FIG. V

opposite is equal to 1.96347" and the side adjacent is 7.875"?

Rule No. 1: Tangent $= \frac{\text{side opposite}}{\text{side adjacent}}$.

1.96347 ÷ 7.875 = .24933. Upon looking in the table of tangents, we find that tangent .24933 is equivalent to 14 degrees (Ans.).

Problem No. 2: What is the length of the side opposite in Fig. V if the angle is 14 degrees and the side adjacent is 7.875"?

Rule No. 2: Side opposite = side adjacent × tangent.
Side adjacent = 7.875". Tangent of 14 deg. = .24933.
7.875" × .24933 = 1.96347" = side opposite (Ans.).

Problem No. 3: What is the length of the side adjacent in Fig. V if the side opposite is 1.96347″ and the angle is 14 degrees?

Rule No. 3: Side adjacent $= \dfrac{\text{side opposite}}{\text{tangent}}$.

Side opposite = 1.96347″. Tangent of 14 deg. = .24933.
∴ 1.96347″ ÷ .24933 = 7.875″ = side adjacent (Ans.).
(Note that the value of tangents range from zero (0) to infinity.)

Uses of Inverse Functions

It frequently happens that an inverse function may be used to advantage, for example, in Fig. VI, if it is desired to find the length of face (or hypotenuse) of a cone for a clutch, the following formula probably would be used:

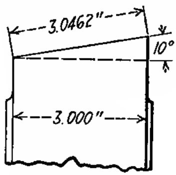


FIG. VI

$$\text{Hypotenuse} = \frac{\text{side adjacent}}{\text{cosine}} = \frac{3''}{.98481} = 3.0462''\ \text{(Ans.)}.$$

This would be a rather long and tedious operation, and since the **Secant** is the reciprocal of the cosine, the process could have been shortened by multiplying the side adjacent (3″) by the secant of 10 deg. (1.0154) which equals 3.0462″ (Ans.).

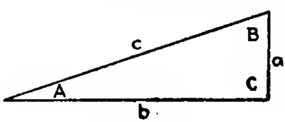


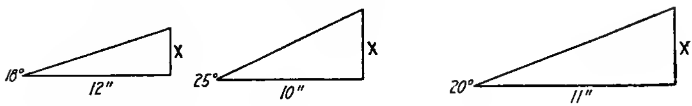
FIG. VII

Solution of Right-angled Triangles

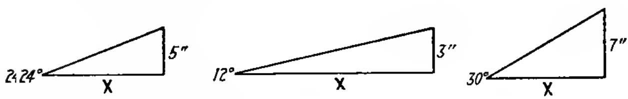
Sides and Angles Known	Formulas for Sides and Angles to Be Found		
Sides c and a.	$b = \sqrt{c^2 - a^2}$	$\sin A = \frac{a}{c}$	$B = 90° - A$
Sides c and b.	$a = \sqrt{c^2 - b^2}$	$\sin B = \frac{b}{c}$	$A = 90° - B$
Sides a and b.	$c = \sqrt{a^2 + b^2}$	$\tan A = \frac{a}{b}$	$B = 90° - A$
Side c. Ang. A. .	$a = c \times \sin A$	$b = c \times \cos A$	$B = 90° - A$
Side c. Ang. B. .	$a = c \times \cos B$	$b = c \times \sin B$	$A = 90° - B$
Side a. Ang. A. .	$c = \dfrac{a}{\sin A}$	$b = a \times \cot A$	$B = 90° - A$
Side a. Ang. B. .	$c = \dfrac{a}{\cos B}$	$b = a \times \tan B$	$A = 90° - B$
Side b. Ang. A. .	$c = \dfrac{b}{\cos A}$	$a = b \times \tan A$	$B = 90° - A$
Side b. Ang. B. .	$c = \dfrac{b}{\sin B}$	$a = b \times \cot B$	$A = 90° - B$

EXERCISES

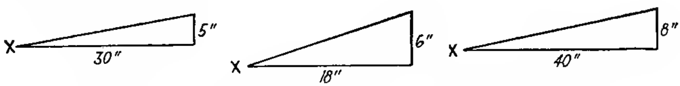
1. Solve for (x):



2. Solve for (x):



3. Solve for (x):



4. Calculating the circumference from root diameter and allowing 5 deg. side clearance on the tool, what will be the actual cutting clearance on the right and left side of a square thread tool used to cut a 4″ root diameter and two threads per inch single thread? (Fig. VIII.)

5. At what angle from regular position will the table of universal milling machine be set to gash a worm wheel meshing with a worm having 4 threads per inch double thread and 2″ P.D.?

6. If in sighting over a 30 deg. triangle, your sight would strike a window in a tower, the base of the tower being 200 ft. away, how high would the window be from the base of triangle?

7. If two public highways join at a certain point, one running at an angle of 32 deg. south of due east and the other 27 deg. north of east, how far would a man walk if he left a point 1 mile from the junction and traveled at an angle of 90 deg. from south road before he came to the north road?

8. If in cutting a taper of 4 degrees included angle, on a piece 10″ long, how far over would you set the tail stock on a lathe?

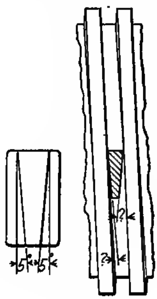
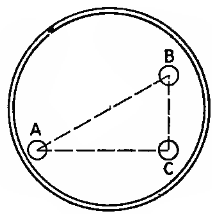


FIG. VIII

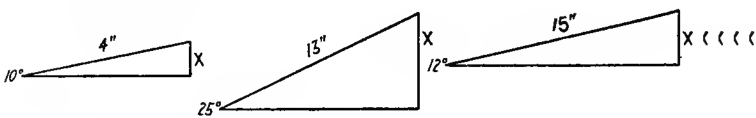


JIG

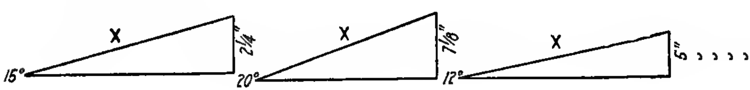
9. A machinist in boring three holes in a jig finds the distance from *A* to *C* is 1″. The hole *B* is at right angles with *A–C* and a line 30 deg. from *A–C* will intersect *B*. What is the distance from *B* to *C*?

10. Using same illustration as in problem No. 9, if the distance from *B* to *C* was 2″, what would be the distance from *A* to *C*?

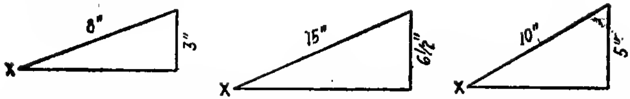
1. Solve for (x):



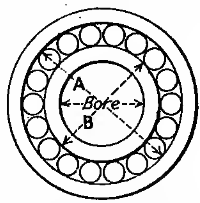
2. Solve for (x):



3. Solve for (x):



4. How many $\frac{1}{2}''$ steel balls will be required for a bearing, allowing 0.002″ clearance between each ball if the inside ball race has a diameter of 1.440″?



A = Outside Diam. of Ball Race
B = Inside Diam. of Ball Race

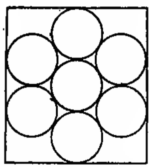
5. What diameter of outside ball race will be required on a bearing, using 15 balls $\frac{1}{4}''$ in diameter, 0.002″ clearance between balls?

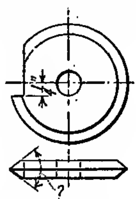
6. The guy ropes of a flag pole make an angle of 35 deg. with the pole. If they extend 30 ft. from the base of the pole, how long are they?

7. What included angle corresponds to a taper of 5.10″ per ft.?

8. If by elevating a transit placed 10 ft. above sea level to an angle of 12 deg., it strikes the top of a tower 4000 ft. away, how high is the tower?

9. What will be the respective dimensions of the sides of a rectangular base that will just fit in a 3″ circle, if the sides are in a ratio of 2 to 1?





10. What is the smallest size rectangular box that can be used to pack seven 1″ steel balls, arranged in a circle?

11. What must be the included angle of a Rivett Dock 8-pitch V threading tool to cut a 60 deg. thread, if the cutter is 3″ in diameter and is ground $\frac{1}{4}''$ below center?

12. From the top of a cliff 2000 ft. high, the angles of depression of two ships at sea are observed to be 45 deg. and 30 deg. If the line joining the ships point directly to the foot of the cliff, find the distance between the ships.

13. Two ships leave the harbor at the same time, one sailing northeast

at the rate of 8 miles per hour, and the other sailing north at the rate of 12 miles per hour. Find the shortest distance between the ships $1\frac{3}{4}$ hours after starting.

14. What is the spiral angle of a spring 3″ in diameter and with a $\frac{5}{8}$″ lead?

15. Fig. I given c 6″, B 15 deg., find A, a and b.

16. Fig. I given b 0.852″, A 14 deg., find a, B and c.

17. Fig. I given c $1\frac{7}{8}$″, b $1\frac{5}{8}$″, find B, A and a.

18. Fig. I given c 12″, A 42 deg., find b, a and B.

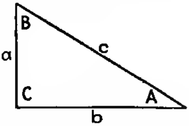


FIG. I

19. In the bushing plate shown in Fig. II, calculate the distance between the following holes: A to B, A to C, C to D, and C to E.

20. In Fig. II, calculate the distances U, W, X, Y, and Z.

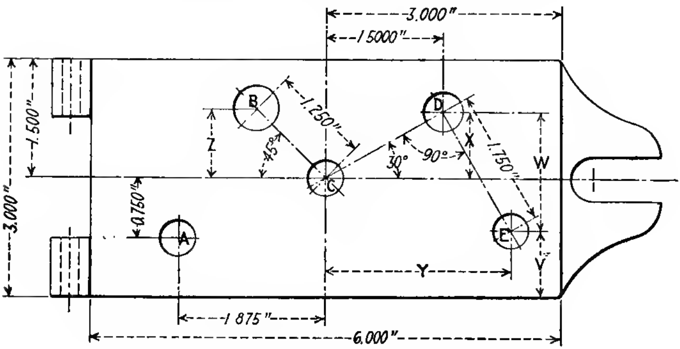


FIG. II. BUSHING PLATE FOR DRILL JIG

Speeds and Feeds

The proper peripheral speeds and longitudinal feeds are necessary in machine tool operations to obtain the maximum production without heating the cutting tools to a softening point.

By **Peripheral Speed** is meant the distance through which a point on the periphery of the work or tool would pass in a given length of time, and is usually designated in feet per minute (f.p.m.).

The peripheral speed of a drill may be determined by multiplying the circumference in feet by the number of r.p.m.

In all turning operations on a lathe, there are three very important questions to be considered: the cutting speed, the feed of the tool, and depth of cut.

The r.p.m. for any work on a lathe can be found by multiplying cutting speed by 12 and dividing the product by the circumference of work in inches.

Cutting Speed is found by multiplying r.p.m. times circumference in inches and dividing by 12.

The cutting speed on a milling machine = circumference of cutter times r.p.m.

The peripheral speed of grinding wheels = circumference of wheel in feet × r.p.m.

Approximate Formulas

To find the revolutions per min., if a certain cutting speed is wanted,

$$\text{r.p.m.} = \frac{\text{cutting speed} \times 12}{\pi \times \text{diameter}}$$

or

$$\text{r.p.m.} = \frac{3.82 \times \text{cutting speed}}{\text{diameter}}.$$

To find the cutting speed in ft. per min. if r.p.m. is given,

$$\text{cutting speed} = \frac{\pi \times \text{diameter} \times \text{r.p.m.}}{12}.$$

To find the total time to finish cut.

$$\text{Total time to finish cut} = \frac{\text{length in inches}}{\text{r.p.m.} \times \text{feed per rev.}}.$$

Chart Showing Approximate Cutting Speeds in Feet per Minute for Various Machines and Materials

Material	Machine	High Speed Steel Tools	Tool Steel Tools
		Speed in Feet per Minute	Speed in Feet per Minute
Tool steel	Drill press	50–60	20–30
	Lathe	50–70	25–35
	Miller	50–60	20–30
	Shaper	40–50	20–25
	Gear cutter	.....	.....
	Planer	40–50	20–50
	Screw machine	60–70	25–35
Cast iron	Drill press	100–170	40–80
	Lathe	75–175	40–80
	Miller	100–150	60–80
	Shaper	80–100	50–60
	Gear cutter	60– 80	30–50
	Planer	70– 90	40–50
	Screw machine	100–150	50–70
Machine steel	Drill press	100–120	50–60
	Lathe	100–150	50–70
	Miller	100–125	50–70
	Shaper	60–80	50–60
	Gear cutter	60–80	30–40
	Planer	50–70	40–50
	Screw machine	100–150	50–70
Brass, bronze	Drill press	200–300	100–150
	Lathe	150–300	70–150
	Miller	150–250	80–125
	Shaper	100–120	60–80
	Gear cutter	100–125	50–60
	Planer	90–100	60–70
	Screw machine	200–300	100–150
Aluminum	Drill press	200–300	100–150
	Lathe	200–300	100–150
	Miller	200–350	100–175
	Shaper	125–200	80–125
	Gear cutter	150–200	70–100
	Planer	150–200	75–100
	Screw machine	200–300	100–150

The above speeds should be increased or decreased according to the nature of the work, tool, lubricant, machine, etc.

EXERCISES

1. What is the peripheral speed of a 1″ drill traveling 110 r.p.m.?

2. In drilling a piece of brass, the spindle makes 3500 r.p.m., and a 1/16″ drill is used; what is the peripheral speed of the drill?

3. In drilling cast iron, using a 1″ drill taking a 0.007″ feed and 130 r.p.m. how long would it take to drill through a piece $6\frac{1}{4}$″ thick?

4. How long will it take to make a cut on a piece 10″ long, in a lathe with a 0.020″ feed running at 180 r.p.m.?

5. Find r.p.m. if a cutting speed of 50 ft. per min. is wanted and the diameter of cutter is 6″.

6. Find cutting speed of a 6″ diameter cutter which runs 60 r.p.m.

7. How long would it take to shape off $\frac{1}{2}$″ of the surface of a piece 6″ long by 4″ wide, feed 0.010″, cut $\frac{1}{8}$″ deep, and it takes 2 sec. to complete the stroke?

8. How many r.p.m. would a 6″ diameter emery wheel on a surface grinder make, whose peripheral speed is 6000 ft. per min.?

9. At what speed should a $\frac{1}{2}$″ drill be run to have a cutting speed of 50 ft. per minute?

10. At what speed should a $\frac{7}{8}$″ end mill be run to obtain a cutting speed of 80 ft. per minute?

11. What is the cutting speed of a lathe tool if the work is 3″ in diameter, running at 100 r.p.m.?

12. What is the cutting speed of a milling cutter 8″ in diameter, running at 60 r.p.m.?

13. How long will it take to finish a cut 18″ long with a 1″ end mill, running at 80 ft. per min., with a 0.010″ feed?

14. How long will it take to make a cut on a shaft 14″ long, using a feed of 0.080″ on a lathe running at 120 r.p.m.

15. What is the time required to drill a $\frac{5}{8}$″ hole 1″ deep in 1000 brackets if a feed of 0.010″ is used, and the drill runs 300 r.p.m. providing it takes 10 seconds to load the jig?

16. How long will it take to cut a keyway 8″ long in 750 shafts, if the cutter runs 240 r.p.m., with a 0.018″ feed, allowing 30 seconds time for loading fixture?

17. What speed should a gear cutter run that is $3\frac{1}{2}$″ in diameter to have a surface speed of 90 ft. per minute?

18. What is the cutting speed of a lathe tool if the work is $2\frac{1}{4}$″ in diameter running at 185 r.p.m.?

Cost Calculation

In large plants, before a machine is manufactured each individual part is classified separately by the planning department and a **Tool Operation Index Sheet** is made out as shown.

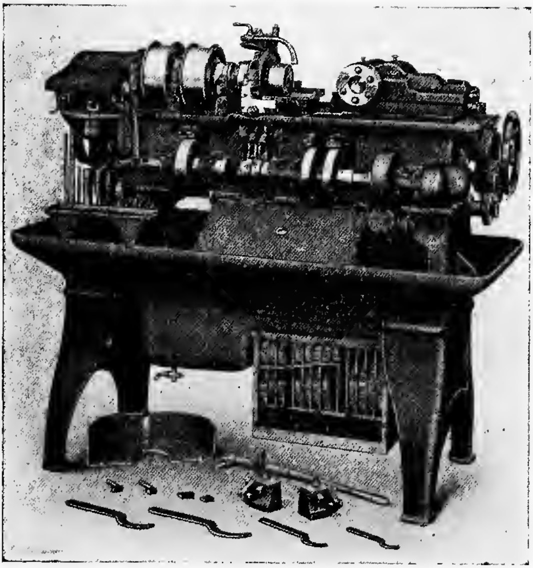
These operations are so arranged as good judgment and past experience shows will be the best and most satisfactory way of handling the particular part in view.

This sheet gives the tool designer the sequence in which the various operations are to be performed and he can thus design his tools and fixtures accordingly. This sheet also gives the necessary machines required and the order in which they should be arranged in the department to facilitate production and prevent any unnecessary waste of space and trucking.

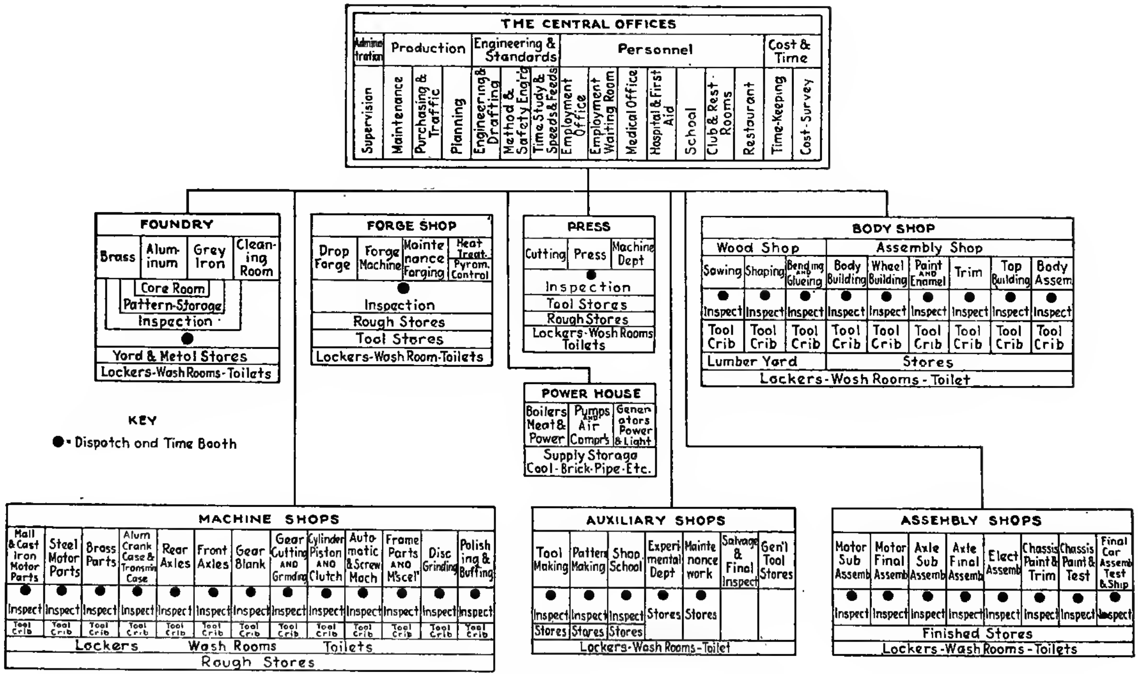
When things are running smoothly in the shop a **Motion Study Record,** as shown, is taken by the time study department of each operation by means of a series of close observations timed by a stop watch. The duration of this study should be sufficiently long enough to get fair average results from which bases the piece work price is determined. This price usually holds good as long as the method, machine, tools or other working conditions are not changed.

The motion study record is followed by an **Instruction Card** which gives to the foreman, set up man and the operator, all instructions relative to tools, jigs and fixtures necessary to perform an individual operation. Also giving time allowance predetermined by a time study, the individual time for each elemental operation, feeds and speeds, etc. The foreman should see that this card is followed out in detail unless a better or more efficient way is found.

For the foreman's information a **Production Routing Sheet** is sent him from the planning department, which gives to him in a classified form the various operations performed in his department, with the piece price of each operation and other information which he should keep on hand for future reference.



Automatic Screw Machine



OUTLINE FOR AN AUTOMOBILE PLANT

TOOL OPERATION INDEX SHEET

Part Name Piston **Material** Cast iron **Part No.** 1024

Oper. No.	Operation	No. Req'd	Name of Mach.	Size	Mach. No.
1	Inspect casting for hard spots, cracks, etc.	1	Bench	10 ft.	√
2	Rough turn diam., face ends and rough groove	2	Potter & Johnson piston mach.	....	409 411
3	Anneal and season	1	Std. gas furnace	No. 636	1108
4	Water test	1	Special fixture	....	1097
5	Drill and mill inside faces of bosses	1	Garvin horizontal drill press	2 spindle	1048
6	Bore and face open end	1	Warner & Swasey hand screw mach.	No. 4	971
7	Center end, finish face end, rough and finish grooves	2	Potter & Johnson	5A	189 200
8	Finish mach. wrist pin hole	1	Koefer special drill press	Special	991
9	Roll piston ring grooves, chamfer grooves and both ends of piston	1	Reed lathe	14″	124
10	Drill oil grooves	1	Leland & Gifford drill press	1 spindle	781
11	Drill, spot face and tap lock screw hole	1	Leland & Gifford drill press	2 spindle	544
12	Finish grind diam.	1	Norton	10 × 36	234
13	Inspect	1	Bench	10 ft.	√
14	Grind relief	1	Norton	10 × 36	236
15	Grind wrist pin hole	2	Bryant	No. 18	1102
16	Cut off center and face end	1	P. & W. shaving mach.	6″	337
17	Polish end	1	Speed lathe	6″	675
18	Inspect	1	Bench	10 ft.	√
19	Oil and place in cartons	1	Bench	5 ft.	√

JOHN DOE MOTOR CO. **Sheets** √ **Page** √ **MOTION STUDY RECORD**

Date Oct. 30, 1919.
Part Name Inlet valve stem guide.
Operation Rough turn .860–.870 and .720–.725 diameters and rough drill and ream .435–.437 hole, turn taper and cut to length.
Mach. No. 300 **Name** No. 4 Warner and Swasey hand screw machine.

Serial No. T20460
Part No. 13047
Oper. No. 8
Dept. T20

No.	Elemental Description	Cutting Speed	Feed	Cuts			Time		Time		Time		Time		Time		Summary	
				L'gth	D'pth	Limit	Cont.	Indv.	Cont.	Indv.	Cont.	Indv.	Cont.	Indv.	Cont.	Indv.	Aver-age	Net
1	Place piece in collet, secure and change feed gears						.15	.15	.16	.16	.14	.14	.15	.15	.15	.15	.15	.15
2	Index turret, bring to work and engage feed.						.18	.03	.19	.03	.17	.03	.18	.03	.18	.03	.03	.03
3	Turn .860–.870 and .720–.725 diameter and center end	200	.020	$3\frac{7}{8}''$	$\frac{5}{32}''$		1.13	.95	1.11	.92	1.12	.95	1.12	.94	1.20	.02	.95	.86
4	Index turret, change speed and feed, bring to work and engage feed						1.16	.03	1.14	.03	1.15	.03	1.15	.03	1.23	.03	.98	.03
5	Drill hole 23/64″	1000		4		±.002	1.60	.44	1.57	.43	1.62	.47	1.62	.47	1.70	.47	.46	.40
6	Index turret, bring tool to work						1.63	.03	1.60	.03	1.65	.03	1.65	.03	1.73	.03	.03	.03
7	Ream hole $\frac{3}{8}''$	1000	hand	4		±.001	1.70	.07	1.66	.06	1.69	.04	1.70	.05	1.77	.04	.05	.05

No.	Elemental Description	Cutting Speed	Feed	Cuts			Time		Time		Time		Time		Time		Summary	
				L'gth	D'pth	Limit	Cont.	Indv.	Cont.	Indv.	Cont.	Indv.	Cont.	Indv.	Cont.	Indv.	Aver-age	Net
8	Index turret, bring tool to work.............						1.73	.03	1.69	.03	1.72	.03	1.73	.03	1.81	.04	.03	.03
9	Form taper............	1000	hand				1.79	.06	1.76	.07	1.77	.05	1.78	.05	1.86	.05	.06	.03
10	Index turret for 1st operation................						1.83	.04	1.79	.03	1.80	.03	1.80	.02	1.89	.03	.03	.02
11	Cut to length..........	1000	hand		$\frac{1}{4}$		1.89	.06	1.84	.05	1.86	.06	1.85	.05	1.96	.07	.07	.07
12	Run back parting tool, loosen collet, take out piece and place in box						1.95	.06	1.89	.05	1.92	.06	1.91	.06	2.03	.07	.07	.05
13																		
14																		
15																		
16																		
17																		
18																		

Base Rate 70 cents **Piece Price:—Man** .0227 **Helper** √ **Girl** √ **Total Actual Time** 1.75

Operator's No. T2009 **Helper's No.** √ **Supersedes Serial** 2633 **Date** 2–15–19 **Std. Allowance %** 11 **Time** 19
Observer Jones **Former Actual Time** 4.43 **Former Std. Time** 5.00 **Spec. Allowance %** √ **Time** √
Foreman's Signature Brown **Former Piece Price:—Man** .0416 **Helper** √ **Girl** √ **Std. Time per Piece** 1.94
Checked By P.V.F. **Reason for Change** Castings were hard when former study was taken, at present castings are annealed. **No. Pieces 1 Hr.** 30 8/9
Approved By R.E.J. **No. Pieces 9 Hrs.** 278
Remarks

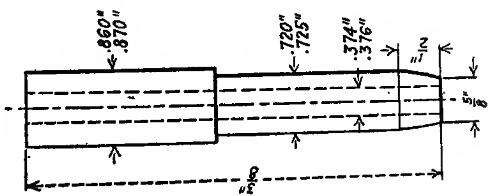
Note.—Sketch and Detailed Description of Tools on Reverse Side of Sheet

Tool Description per Each Sub-operation

Oper. No.	Tool No. or Size	Material	Other Descriptive Details
1	394	T.S.	Bell center for centering stock while chucking.
2	478	T.S.	Box tool for turning O.D. and centering end.
3	23/64	H.S.S.	Drill.
4	.375	H.S.S.	Reamer.
5	$\frac{1}{8} \times 1 \times 6$	H.S.S.	Cutting off tool.
			Snap gauge for large diameter.
			Snap gauge for small diameter.
			Gauge for over all length.
			Plug gauge for hole.
9	798	T.S.	Box tool for forming taper.

Fixture No. √ **Or Description** √
Tool Endurance—Hours 9 **Approx. No. of Pieces** 278
Belts—Length 19′ **Width** 3″ **Ply** 3
Kind of Material Cast iron **Weight** ½ lb. **Condition** Soft **Lubricant** None

Sketch



Turn, drill, ream and form taper

Remarks:
To be finish turned on mandrel.

7

INSTRUCTION CARD | **JOHN DOE MOTOR CO.**

Dept. T20 | **Symbol** 12046
Part Name Crank case (lower half) | **Serial No.** T20124
Operation (500) Mill oil pump pad. | **Mach. Name** No. 2 Cinn. Miller
Material Lynite **Mach. No.** 73 | **Date** 9-15-19
Rate per Piece:—Oper. .084 **Helper** .076 **Base Rate:—Oper.** 70 **Helper** 60
Note: All Time Expressed in Minutes and Decimals. | **Approved By** J. J. Jones

METHOD. To be Strictly Followed	Mach. Speed R.p.m.	Feed in 1000″	Time Gross	Time Net	Equipment
Clean fixture			.15	.15	Fixture T683.
Take case from rack and place on fixture			.35	.35	General use cutter.
Secure case on fixture			.65	.65	No. 854.
Check seat (use feeler)			.10	.10	8″—16 blades.
Place water guard in position			.09	.09	½″ socket wrench.
Start machine, position cutter to work and engage feed			.14	.14	Gauge No. 179.
Rough mill pump pad	120	.030	1.15	1.15	8″ brush.
Change feed lever			.20	.20	1½ lb. lead hammer.
Run table back to clear cutter			.16	.16	
Set to graduation for finish mill			.25	.25	
Position cutter to work, engage feed			.09	.09	
Finish mill pump pad	120	.075	.45	.45	
Set graduation to position for rough mill			.25	.25	
Stop machine and run table back to clear cutter			.17	.17	
Remove water guard			.09	.09	
Release case from fixture			.35	.35	
Remove case and place in rack			.20	.20	
Net time				4.84	
30% Allowance				1.45	
PCS. in 9 Hrs.			**Standard Time**	6.29	**Issued By** Brown

Remarks: Supersedes study serial No. T202B.

PRODUCTION ROUTING SHEET

Part Name Crank case **Material** Aluminum **Model** 53 **Part No.** 12046
Date 11–27–18 **Cancels Issue No.** C–78 **This Issue No.** E107 **Sheet No.** √ **Book No.** 2

Oper. No.	Jig No.	Mach. No.	No. of Gangs	Operation	Operator	Helper	Schedule No.	Dept.	Location	Piece	Price	Production per Hr.
1	1048	300 and 784	2	Machine ream main bearing seats	1	1	T20473	T20	1–C–10	O. H.	.09 .081	9
2	3570	109 and 78	2	Martell ream main bearing seats	1	1	T20124	"	1–C–10	O. H.	.24 .22	$3\frac{1}{2}$
3	901	301	1	Face thrust bearing to width	1	1	T20203	"	1–C–11	O. H.	.078 .070	11
4	977	√	1	Hand ream cam shaft, generator shaft and cam shaft holes	2	√	T20107	"	1–C–12	O. O.	.23 .23	4
5	948	307	1	Machine ream water pump and oil pump pads	1	1	T20475	"	1–C–12	O. H.	.09 .08	9
6	406	981	1	Face synchronizer end	1	1	T20166	"	1–C–13	O. H.	.11 .10	8
7	378	707	1	Drill and ream lock screw hole and hand face propeller end	1	1	T20244	"	1–C–13	O. H.	.09 .08	9
8	√	√	1	Disassemble upper and lower half	2	√	T20109	'	1–C–14	O. O.	.03 .03	25
9	586	1034	1	Drill and counterbore 7 dowel pin holes in lower half	1	1	T20419	'	1–C–15	O. H.	.10 .09	8
10	1057	580	1	Drill and counterbore 7 dowel pin holes in upper half	1	1	T20420	"	1–C–15	O. H.	.10 .09	8
11	√	√	√	Inspection								

Cost Calculations

In estimating the cost of operating a machine shop, the cost of labor, material, overhead expenses, etc., must be considered.

Overhead Expenses include power, heat, light, insurance, depreciation, supervision, etc., and all other expenses which are not classified under labor or material.

The following estimates will be approximately correct for a shop employing 12 men, and doing tool work, such as dies, jigs, gages, fixtures, etc.

Cost of Materials

High carbon steel (tool steel)	12	cents	per	lb.	approximately
High speed steel	90	"	"	"	"
Machine steel (low carbon steel)	5	"	"	"	"
Screw stock (low carbon steel)	7	"	"	"	"
Cold rolled steel (low carbon steel)	5	"	"	"	"
Cast iron-castings	3	"	"	"	"
Bronze	25	"	"	"	"
Brass	17	"	"	"	"
Copper	20	"	"	"	"
Aluminum	60	"	"	"	"
Lard oil	63	"	"	gal.	"
Machine oil	13	"	"	"	"

Cost of Labor

Three first class toolmakers	75	cents	per	hour
Three second class toolmakers	60	"	"	"
Two lathe hands	50	"	"	"
One utility man	40	"	"	"
One apprentice	20	"	"	"

Overhead Expenses

Light	$3.00 per month
Gas for furnace	$4.00 " "
Heat	$5.00 average per month
Power	Approx. 30 cents per hour for 10 h.p.

Supervision { foreman of room....$0.80 per hour
manager...........$1.00 " "
clerk..............$12.00 per week.

Insurance { Accident............$1\frac{1}{2}\%$ of pay roll.
Fire.................$5.00 per $1000 per year.

Rent—$25.00 for room 40′ × 40′
Depreciation on machinery—10%
Interest on money—6%
Cost of investment:

Two lathes—14″ and 18″	$1600
One universal milling machine	1300
Hand milling machine	250
Shaper	350
Speed lathe	75
Gas furnace	75
Bath grinder	800
Two drill presses	250
Miscellaneous tools: drills, reamers, counterbores, etc.	200
Total	$4900

It would require an investment of $5000 for equipment; the payroll for 90 days would be $3000, cost of materials and overhead expenses $2000, also a surplus of $2000 to take care of bad debts and any emergencies, making a required capital of $12,000.

The **Cost of Raw Materials** is usually quoted in pounds, therefore it is necessary to find volume or weight of stock.

Cast iron weighs 0.26 lbs. per cu. in.
Carbon steel weighs 0.28 lbs. per cu. in.
Bronze weighs 0.31 lbs. per cu. in.
Brass weighs 0.30 lbs. per cu. in.

In all operations on machinery in a factory, approximately 10% of time is lost in changing tools, etc. every day.

Depreciation is the natural decrease in the value of equipment.

Maintenance is the amount of money required to maintain the equipment in good working condition.

EXERCISES

1. What would be the cost of a bar of high carbon steel 3″ in diameter 10 ft. long?

2. What would be the cost of a round bar of high speed steel 4″ in diameter by 10 ft. long?

3. What would it cost for material to make 24 cast iron spur gears 1″ wide, outside diameter 4″, allowing $\frac{1}{8}$″ for cutting off tools and $\frac{1}{8}$″ for finishing over all. (Cast iron weighing 0.26 lbs. per cu. in.)

4. Find the cost of a 1″ round piece of brass 12 ft. long estimating brass at 0.30 of a lb. per cu. in.

5. What would a piece of high carbon steel 4″ × 4″ × 10 ft. long cost?

6. What would it cost for material for 12 die blocks of high carbon steel $1\frac{1}{4}$″ thick, 6″ wide by 8″ long?

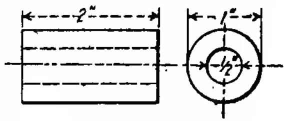
7. What will be the cost of a taper piece of C.R.S. 10″ long, 3″ in diameter at the large end and 1″ in diameter at the small end?

8. Find the cost of a 1″ hex. brass bar 16 ft. long.

9. Find the cost of 6 pieces of C.R.S. $\frac{1}{8}$″ × 4″ × 12″.

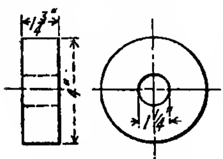
10. What will be the cost of 100 pieces of high speed steel $\frac{1}{4}$″ × $\frac{3}{8}$″ × $2\frac{1}{2}$″?

11. Determine cost of material and machine operations on 60 tool steel bushings, finished size 1″ diameter × 2″ long × $\frac{1}{2}$″ hole, rough size stock $1\frac{1}{16}$″ diameter allowing $\frac{1}{8}$″ for cut off. Turn with one cut at 0.004″ feed per rev., and 150 r.p.m. Drill at 0.005″ feed and ream at 0.015″ per rev., allowing 30 sec. for facing and cutting off. Lost time on each piece 5 sec. for change of turret, etc., wages $37\frac{1}{2}$ cents per hour.



BUSHING

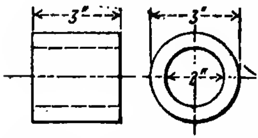
12. What will it cost to machine on a turret lathe, 100 spur gear blanks made of cast iron 4″ outside diameter, $1\frac{1}{4}$″ hole, $1\frac{3}{4}$″ wide, running at 70 r.p.m., using a feed of 0.010″ per rev. for turning outside diameter, 0.005″ feed for drilling and boring, and 0.020″ feed for reaming, allowing 2 sec. for revolving turret between operations, wages $37\frac{1}{2}$ cents per hour.



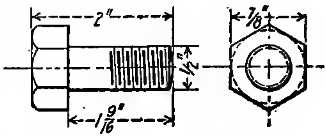
GEAR BLANK

13. Estimate cost of cutting 30 teeth on each gear in the preceeding problem, on a B. & S. gear cutter, if it requires 14 sec. to make each cut and 1 sec. for return and indexing, allowing 1 minute to change blanks, wages 35 cents per hour.

14. Determine cost of machining and material, doing work on a lathe for a bronze bushing 3″ outside diameter, 3″ long and 2″ hole finish size, rough size $3\frac{1}{8}$″ outside diameter and $3\frac{1}{8}$″ long, $1\frac{1}{2}$″ cored hole, spindle speed 150 r.p.m., chucking 30 sec., feed for drilling 0.005″ per rev., for rough and finish boring 0.005″ feed, reaming 0.020″ feed, placing on arbor 60 sec., rough and finish turning on outside at 0.006″ feed, for facing off each end 20 sec., allowing 50 sec. for changing each tool, wages $37\frac{1}{2}$ cents per hour.

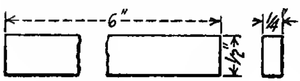


15. Estimate cost of material and machining on Acme automatic screw machine for making 10,000, $\frac{1}{2}$″ hexagon head screws 2″ long, body $1\frac{9}{16}$″ long. It requires 30 sec. to do all operations on one screw. The four operations being done at the same time, allowing $\frac{1}{8}$″ for cutting off; wages 45 cents per hr., one man running 3 machines and $\frac{7}{8}$″ C.R.S. hex. stock is used.



$\frac{1}{2}$″ × 12-Hex. Head Screw

16. What will it cost to make 10 pair of tool steels parallels $\frac{1}{4}$″ × $\frac{1}{2}$″ × 6″, finish size, rough size $\frac{5}{16}$″ × $\frac{9}{16}$ × $6\frac{1}{8}$″. Take one cut with a 0.005″ feed per stroke overall for finishing on shaper. Leaving 0.016″ overall for grinding and using a cross feed of 0.010″ and 0.002″ deep per cut on a surface grinder. It requires 6 sec. for a cut and return for grinder, and 3 seconds for shaper, allowing 3 min. for shaping off each end, and 4 min. for grinding each end. Lost time—1/10, wages 35 cents per hour.



Parallel. 20 Wanted

Levers

A **Lever** is an inflexible rod capable of motion, about a fixed point, called a **Fulcrum.** The rod may be straight, curved or bent at any angle.

There are 3 kinds of levers, or in other words 3 arrangements of the force, weight and fulcrum.

In the **Lever of the First Class,** the fulcrum lies between the points at which the force and load act. Fig. I.

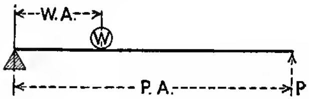


Fig. I

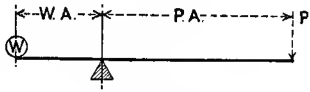


Fig. II

In the **Lever of the Second Class,** the load acts at a point between the fulcrum and the force. Fig. II.

In the **Lever of the Third Class,** the action of force is between the load and the fulcrum. Fig. III.

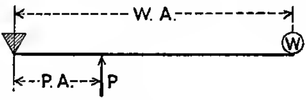


Fig. III

Levers are usually used to gain power at the expense of time, thus, in a first class lever, if the distance from the fulcrum to power is 5 times the distance to the weight, it will give 5 times the power, but it will take a movement 5 times greater than the weight moves.

Levers of the 3d class involve a mechanical disadvantage, as the power must always be greater than the weight.

Law of Levers. The power multiplied by its distance from the fulcrum is equal to the weight multiplied by its distance from the fulcrum.

The law for bent levers (Fig. IV and Fig. V) is the same as for straight levers, but the length of arms is computed on lines from the fulcrum at right angles to the direction in which the power and weight act.

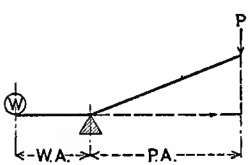


Fig. IV

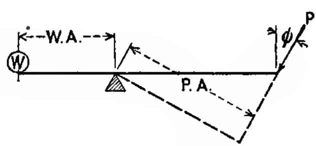


Fig. V

Formulas:

P = Power or force.

W = Weight or resistance.

PA = Power arm or distance from fulcrum to point where power is applied.

WA = Weight arm or distance from fulcrum to point where weight or resistance is applied.

Thus, the law of the levers becomes $P \times PA = W \times WA$.

$$P = \frac{W \times WA}{PA}, \qquad W = \frac{P \times PA}{WA},$$

$$PA = \frac{W \times WA}{P}, \qquad WA = \frac{P \times PA}{W}.$$

Example: What force 15″ from fulcrum will balance a weight of 300 lbs. 5″ from fulcrum?

$$P = \frac{W \times WA}{PA}, \qquad P = \frac{\overset{100}{300} \times 5}{\underset{3}{15}} = 100 \text{ lbs. (Ans.)}.$$

EXERCISES

1. What force 8 ft. from the fulcrum will balance a weight of 500 lbs. 10″ from the fulcrum in a lever of the 1st class? Of 2d class?

2. When a weight of 426 lbs. is balanced on the end of a lever of the 1st class, by a force of 60 lbs. $2\frac{1}{2}$ ft. from fulcrum, what is the distance from weight to fulcrum?

3. What force is required to raise a 1 ton weight, using a lever of the 2d class, if the W.A. = 24″ and P.A. = 10 ft.

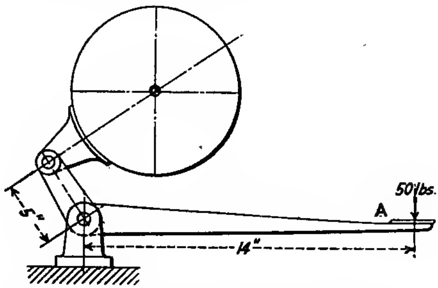
4. Find the weight that a force of 270 lbs. will lift with a lever of the 3d class if P.A. = 4 ft. and W.A. = 10 ft.

5. What must be the length of W.A. to lift a 150 lb. weight with a lever of the 1st class if P.A. = 6 ft. and power = 50 lbs.?

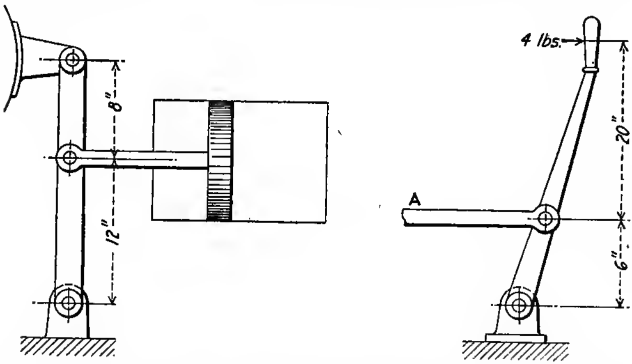
6. What must be the length of W.A. if a force of 750 lbs. 22″ from fulcrum lifts a weight of 1000 lbs. in a 2d class lever?

7. In a lever of the first class what weight will a 100 lb. force acting 4 ft. from fulcrum at an angle of 30 deg. lift, if the weight is 20″ from the fulcrum?

8. What will be the force in lbs. applied at the brake shoe, if a pressure of 50 lbs. is applied with a foot at the end of arm "*A*"?



9. What will be the force in lbs. exerted by rod "*A*," if a person pulls with a 40 lb. force on the lever as shown in sketch?



10. If an air brake lay-out is as per sketch, what will be the force in lbs. exerted against the brake shoe, if the air cylinder is 10" in diameter and the air pressure is 100 lbs. per sq. inch?

Pulleys

A **Pulley** is a wheel mounted to revolve on an axis and having a grooved rim in which a cord, band or chain is passed to transmit the force applied in another direction.

A **Pulley Block** is a device for holding one or more pulleys as a unit.

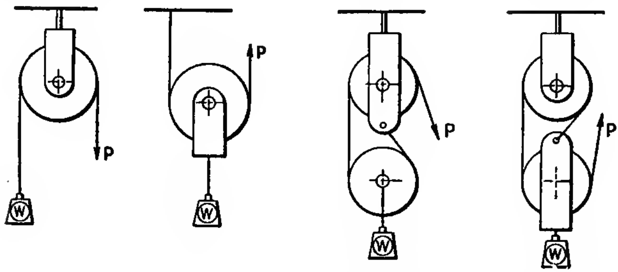


FIG. I FIG. II FIG. III FIG. IV

Pulleys are either **Fixed** or **Moveable,** depending upon whether they are held in a fixed position or move with the given load.

Fixed pulleys are those that have a fixed block Fig. (I) and are generally used to change the direction of the power applied.

Moveable pulleys are those that have movable centers (Fig. II).

Fig. III shows a combination of a fixed and movable pulley. In this arrangement, in order for the weight (W) to move 1 ft. the power (P) must move through a distance of 2 ft., thus $W = 2P$.

Whenever possible the pulleys should be so arranged that the pull would come in the direction the weight is to be moved, as shown in Fig. IV.

Rule for Pulleys.—The force (P) multiplied by the number of strands (N) from the movable pulley, will equal the weight (W) that can be raised, or $P \times N = W$.

Thus, if a force of 1 lb. is exerted in Fig. III, a 2 lb. weight would be raised $P \times N = W$. $1 \times 2 = 2$ (Ans.).

If a 1 lb. force is exerted at P in Fig. IV, a 3 lb. weight would be raised. $P \times N = W$. $1 \times 3 = 3$ (Ans.).

Fig. V shows a **Differential Pulley** arrangement used extensively in the machine shop. In this form of pulley an endless chain replaces the rope. The two pulleys at the top are of slightly different diameters, but rotate together as one piece. In operation, as the chain is drawn over the large wheel, it passes around the lower pulley, up over the small wheel from which it is unwound, causing the loop in which the movable pulley rests to be shortened by an amountequal to the difference in circumference of the two upper wheels, when they have made one revolution. This would cause the weight to raise one-half of this amount.

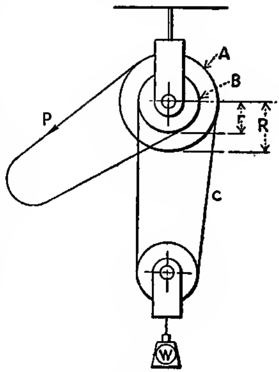


FIG. V

Example: In Fig. V the two upper pulleys are respectively 16″ and 15″ in circumference. As the power applied moves through a distance of 16″ the small pulley will unwind 15″ of chain, causing a shortening of the loop *c* of 1″, which will raise the weight (*W*) ½″, giving a ratio of load to power of 32 to 1.

Rule.—$P = \frac{W(R - r)}{2R}$. $W = \frac{2PR}{R - r}$.

EXERCISES

1. How many movable pulleys would be required to balance 100 lbs. with a 50 lb. weight?

2. Determine weight required to balance a 1 ton weight, using 5 movable pulleys arranged as in Fig. IV?

3. What power is required to raise a weight of 150 lbs., using pulley arrangement in Fig. II?

4. Using pulley arrangement in Fig. III?

5. Using pulley arrangement in Fig. IV?

6. What weight can be raised with a force of 80 lbs., if 3 loose pulleys are used, arranged as in Fig. III?

7. What power will be required to raise a weight of two tons with a differential pulley, if the diameter of the two upper pulleys are 19″ and 20″ respectively?

8. What weight can be raised with a force of 100 lbs. with a differential pulley, if the two upper wheels have 18 and 17 teeth respectively, 2 pitch, (distance from one tooth to another = 1.5708″), allowing 20% for friction?

9. Using a differential block with pulley 15″ and 12″ in diameter, what force is required to raise a 250 lb. casting where there is a friction loss of 15%?

10. In a differential block with pulley 10″ and 15″ in diameter, a pull of 90 lbs. is required to raise a weight of 500 lbs. How much force is used up in overcoming friction?

Screws

A **Screw** is a modified form of inclined plane. The lead of the screw or the distance the thread advances in going around the screw once being the height of the incline, and the distance around the screw measured on the thread being the length of the incline.

When a force is applied to raise a weight or overcome resistance by means of a screw or nut, either the screw or the nut may be fixed, the other being movable.

The force is generally applied at the end of a wrench or lever arm, or at the circumference of a wheel.

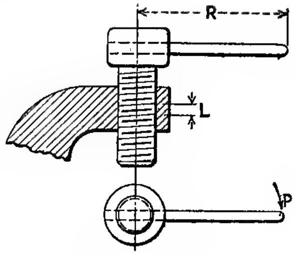
The ratio of the power to weight is independent of the diameter of the screw.

In actual work a considerable proportion of the power transmitted is lost through friction.

Rule.—The force applied multiplied by the circumference of the circle through which the force arm moves, equals the weight or resulting force multiplied by the lead of the screw in inches.

Formulas

$P : W :: L : 2\pi R.$



P = Power applied.
L = Lead of screw.
R = Length of bar, wrench or radius of hand wheel used to operate the screw.
W = Resulting force or weight moved.

$$P = \frac{W \times L}{2\pi R}, \qquad W = \frac{P \times 2\pi R}{L}.$$

EXERCISES

1. What weight can be raised with a 4 pitch jack screw, if a force of 75 lb. is applied to a lever 15″ long?

2. What pressure can be obtained with a $\frac{1}{2}$″ × 12 screw, if a 25 lb. force is applied at the end of a wrench 8″ long?

3. What length of wrench is required to obtain a pressure of 11310 lbs. on a 1″ × 8 screw, if a 25 lb. pressure is exerted on the end of the wrench?

4. If a pressure of 125 lbs. is applied to a 14″ lever on a 4 pitch jack-s crew, estimate pressure if 20 percent is lost through friction.

5. Estimate pressure produced on a milling machine vise if the screw has 6 threads per inch, length of handle 10″ and pressure applied is 75 lbs. loss through friction 30 percent.

6. Estimate pitch of thread required to give a pressure of 3770 lbs., if a 25 lb. pressure is applied to a lever 12″ long.

7. Determine leverage required for a screw of 8 pitch, to give a pressure of 5000 lbs. at end of screw, if a pressure of 25 lbs. is applied at end of lever, and loss through friction is 30 percent.

8. How many jack screws must be used to raise a machine weighing 14844 lbs., if the screws have a $\frac{1}{3}$″ lead, $10\frac{1}{2}$″ lever and 25 lb. pressure applied to lever, allowing 25 percent loss through friction?

Inclined Planes

The **Inclined Plane** is a flat surface sloping or inclined from the horizontal. A body moving up an inclined plane

as opposed both by gravity and friction, while one moving down an inclined plane is assisted by gravity and is opposed by friction only.

When the force which is being applied, is exerted in a direction parallel to the inclined surface, as in Fig. I, it is evident that the power must move through the distance equal to the incline in order to raise a weight the desired height. The gain in power will then be equal to the length of the incline divided by the height.

Rule.—$P : W :: H : L.$

$$P = \frac{W \times H}{L}. \qquad W = \frac{P \times L}{H}.$$

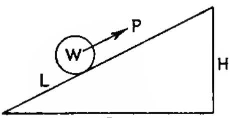


Fig. I

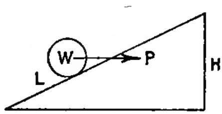


Fig. II

If the force acts along a line parallel to the base B, as in Fig. II, then $P : W :: H : B$.

$$P = \frac{W \times H}{B}. \qquad W = \frac{P \times B}{H}.$$

If the force acts at any angle to the plane as X in Fig. III, then $P : W :: \sin Y : \cos X$.

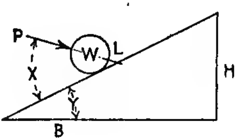


Fig. III

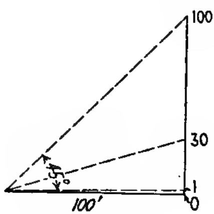


Fig. IV

$$P = \frac{W \times \sin Y}{\cos X}. \qquad W = \frac{P \times \cos X}{\sin Y}.$$

Grade Percent or percent grade is the ratio of the height or elevation to the space covered, *i.e.*, if a road elevation rises 1 ft. in 100 ft. it is called a 1% grade. If a road rises 30 ft. in 100 ft. it is a 30% grade. If it rises 100 ft. in 100 ft. it is called a 100% grade, or equivalent to an angle of 45 degrees (Fig. IV).

EXERCISES

(In the following problems, friction will not be considered)

1. In Fig. I, what force is required to roll a 1 ton weight up the incline if B is 18 ft., and H 8 ft.?

2. In Fig. II, what force is required to roll a 1 ton weight up the incline if B is 18 ft., and H 8 ft.?

3. What weight can be drawn up the incline in Fig. I, if power is 1800 lbs., H is 4 ft. and B 20 ft.

4. What weight can be drawn up the incline in Fig. II, if power is 150 lbs., H is 18″, and L is 72″?

5. What power will be required to hold a ball weighing 380 lbs. in Fig. III, if the incline is 27 deg. 26′ from the horizontal, and the force is acting 18 deg. 45′ from the incline?

6. What weight will a force of 200 lbs. acting at an angle of 30 deg. from the horizontal, sustain if the weight is on an incline whose base is 18 ft. and height 4 ft.?

7. What force will be required to pull a 2200 lb. automobile up a 15 percent grade, if the force is applied parallel with the grade?

8. If an automobile going a distance of one mile rises to an elevation of 264 ft. What is the average percent grade of the road?

9. A car with a 122″ wheel base is standing on a road with its front wheels 18.3 inches higher than the rear wheels. What is the grade percent.

10. What is the grade percent of a roller coaster track, if the angle of rise is 24 degrees?

Wedges

A **Wedge** is a pair of inclined planes united at their bases (Fig. I).

The power is usually applied by a blow of a heavy body or by pressure.

Wedges are used for splitting logs and stones, and raising heavy weights short distances. Due to the excessive friction of wedges, they are not very efficient.

Rule.—$P : W :: T : L$.

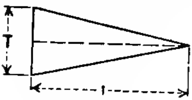


FIG. 1

P = Power applied,
W = Weight or resistance,
T = Thickness of wedge,
L = Length of wedge.

$$W = \frac{P \times L}{T}, \qquad L = \frac{W \times T}{P}, \qquad T = \frac{L \times P}{W}.$$

EXERCISES

(In the following problems, friction will not be considered)

1. What force will be required behind a wedge used to raise a weight of 100 lbs. 3″ high, if the wedge has a sliding motion of 15″?

2. What length wedge must be used to raise a weight of 1 ton with a force of 200 lbs., if the thickness of the wedge is 2″?

3. What weight can be raised with a force of 300 lbs. acting on a wedge 10″ long and $\frac{3}{4}$″ thick?

4. What force will be required to drive a wedge 4″ long and $\frac{3}{8}$″ thick to raise a 50 lb. casting?

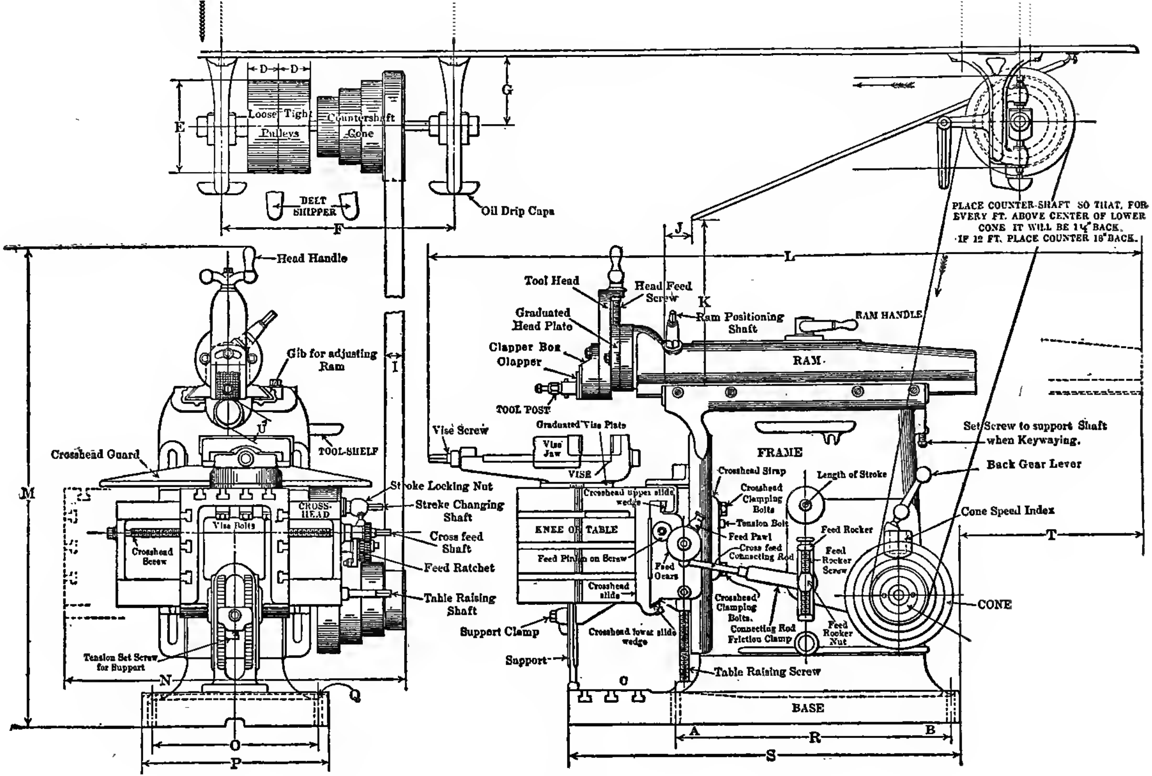
5. What force will be required to drive a wedge 12″ long and $\frac{1}{2}$″ thick into a log that has a resistance of 1800 lbs. against splitting?

Gearing Definitions, etc.

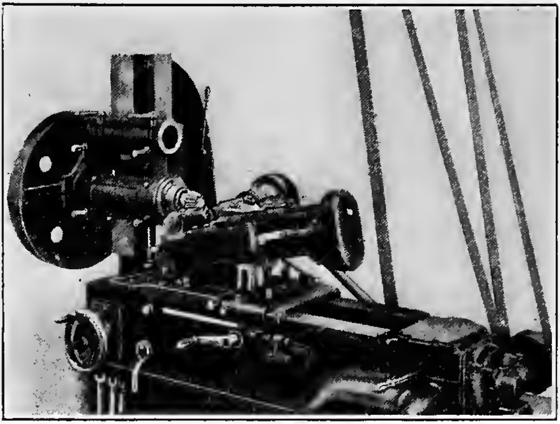
The **Center Distance** of a pair of gears is the shortest distance between the centers of the shafts, on which they are mounted.

The **Pitch Circles** of a pair of gears have the same diameters as a pair of friction rolls which would fill the same center distance and revolve at the same velocity ratio.

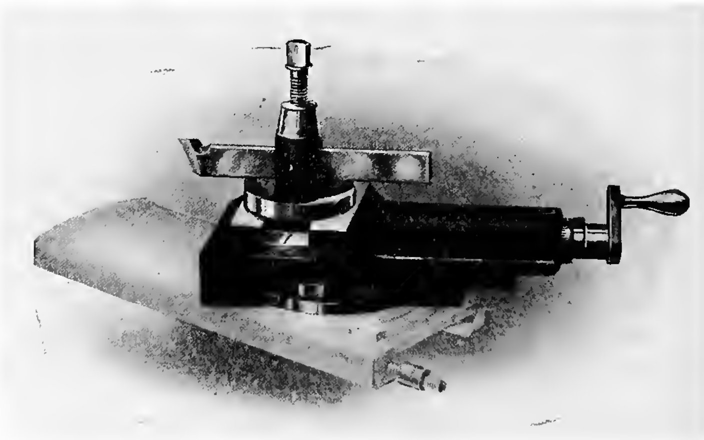
The **Pitch Diameter** of a gear is the diameter of its pitch circle.


Loose Tight
Pulleys
Countershaft
Cone
BELT SHIPPER
Oil Drip Cups
PLACE COUNTER-SHAFT SO THAT, FOR EVERY FT. ABOVE CENTER OF LOWER CONE IT WILL BE 1¼" BACK.
IF 12 FT. PLACE COUNTER 18" BACK.
Head Handle
Gib for adjusting Ram
TOOL-SHELF
Crosshead Guard
Stroke Locking Nut
Stroke Changing Shaft
CROSS-HEAD
Vise Bolts
Crosshead Screw
Cross feed Shaft
Feed Ratchet
Table Raising Shaft
Tension Set Screw for Support
Tool Head
Head Feed Screw
Graduated Head Plate
Clapper Box
Clapper
TOOL POST
Ram Positioning Shaft
RAM HANDLE
RAM
Vise Screw
Graduated Vise Plate
Vise Jaw
VISE
FRAME
Set Screw to support Shaft when Keywaying.
Back Gear Lever
Crosshead Strap
Length of Stroke
Crosshead Clamping Bolts
Tension Bolt
Cone Speed Index
Crosshead upper slide wedge
KNEE OF TABLE
Feed Pawl
Feed Rocker
Feed Pinion on Screw
Cross feed Connecting Rod
Feed Rocker Screw
Feed Gears
Crosshead slide
Crosshead Clamping Bolts.
CONE
Connecting Rod Friction Clamp
Feed Rocker Nut
Support Clamp
Crosshead lower slide wedge
Support
Table Raising Screw
BASE

SHAPER



Automatic Spur and Bevel Gear Cutting Machine



Compound Rest

The **Diametral Pitch** is the number of teeth a gear has per inch of pitch diameter. To find the D.P., divide the number of teeth by the P.D. The P.D. in turn may be found by dividing the number of teeth by the D.P.

The **Circular Pitch** is the distance from the center of one tooth to the center of the next, measured along the pitch line. To find the circular pitch, divide the pitch circle by the number of teeth, or divide π by the diametral pitch.

The **Size of Gear Tooth** is designated by its pitch, thus, a 10 pitch tooth has an addendum of 1/10″ and a dedendum of 1/10″.

The **Tooth Thickness** is measured along the pitch line and is one-half the circular pitch.

The **Addendum** is the height of the tooth above the pitch line.

The **Dedendum** is the depth below the pitch line to which the tooth of the mating gear extends.

The **Working Depth** is the depth in the tooth space to which the tooth of the mating gear extends, and is equal to the dedendum plus addendum.

The **Clearance** is the distance from the point of the tooth to the bottom of the space in the mating gear.

The **Whole Depth** is the distance from the top of the tooth to the bottom of the same tooth and consists of the addendum, dedendum and clearance.

The **Outside Diameter** is found by adding twice the addendum to the pitch diameter.

The **Root Diameter** is the diameter at the bottom of the tooth space.

The **Face** of the gear tooth is that part of the tooth outline which extends above the pitch line.

The **Flank** is that part of a gear tooth outline below the pitch line.

The **Fillet** is the rounded corner where the flank of the tooth runs to the bottom of the tooth space.

The **Base Circle** is the circle from which the involute curve is generated. It is drawn tangent to the pressure line. Its position will vary according to the pressure angle used. The two pressure angles used are the $14\frac{1}{2}$ and 20 deg., the latter being used where a short stubby tooth is required called the "Stub Tooth," while the former is used to the greatest extent. For a $14\frac{1}{2}$ deg. pressure angle tooth gear, the base circle will lie inside of the pitch circle a distance equal to 1/60 of the P.D.

Rotary Motion can be transmitted by belts, chains, shafts, universal joints, friction discs, gearing, etc.

A correctly cut gear will transmit a uniform motion.

The **Principal Styles of Gearing** are spur, herringbone, bevel, spiral or helical and worm.

The two standard tooth curves are the **Involute** and **Cycloidal.**

An **Involute Curve** is most desirable because it will allow a certain amount of variation in the center distance and is used almost universally.

The involute curve is generated by unwinding a string from the base circle and allowing a point on the string to describe a curve, which will be an involute.

The **Cycloidal** will not permit any variation in the center distance and can be generated by two different circles, **Epicycloidal** by revolving a circle on the outside of base and **Hypocycloidal** on the inside.

The ratio of the speeds of two gears that run together is called their **Velocity Ratio,** and is in inverse proportion to their sizes or P.D.

Spur Gears are used to transmit power between two shafts running parallel with each other.

Herringbone Gears conform to two spiral gears, one right hand and the other left hand, fastened to each other, thus equalizing the side thrust. These gears are very quiet in . . .

action, due to some part of the tooth always being in full action.

Bevel Gears are used to transmit power from one shaft to another when the axes are not parallel to each other, but in the same plane.

A bevel gear blank is similar to a frustum of a cone.

Spiral or **Helical Gears** are the same as spur gears, but teeth are cut other than at right angles with the axis.

No. of Cutter	No. of Teeth	No. of Cutter	No. of Teeth
1...........	135 to a rack	5..........	21 to 25
2...........	55 " 134	6..........	17 " 20
3...........	35 " 54	7..........	14 " 16
4...........	26 " 34	8..........	12 " 13

A **Worm** and **Worm Wheel** is used where a greatly reduced speed ratio is desired and consists of a single or multiple thread worm meshing into a worm wheel similar to a concave faced spur whose teeth are cut angular.

The relative efficiency of different styles of gearing are as follows: 1st—spur; 2d—herringbone; 3d—bevel; 4th—spiral or helical; 5th—worm.

For cutting smooth running involute gear teeth, 8 cutters are required for each pitch. These cutters are adapted to cut from a gear of 12 teeth to a rack. The following table shows the list of cutters and the number of teeth it will cut:

FIG. I

Spur Gearing

Spur gears are the most commonly used gears. They are cylindrical in shape and the teeth are cut parallel with the axis.

The different terms commonly used for parts of a spur gear are as per sketch and explanation.

In Fig. II and III

A = Cir. pitch or distance from center of one tooth to next, measured on the pitch line.

B = Clearance.

C = Addendum—top of tooth between O.D. and P.D.

D = Dedendum—bottom of tooth between P.D. and clearance.

E = Whole depth—addendum, dedendum and clearance.

F = Working depth—addendum and dedendum.

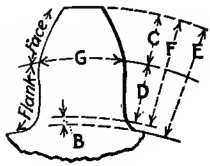


FIG. II

G = Thickness of tooth—width of tooth from outside to outside on pitch line.

H = Outside diameter.

I = Pitch diameter or the diameter of gear from one pitch line to the opposite on center line.

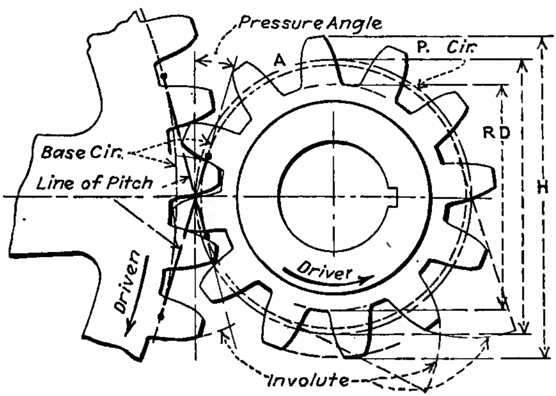


FIG. III

Abbreviations Generally Used

P = Diametral pitch, or pitch.
O.D. = Outside diameter.
N = No. of teeth.
Np = No. of teeth in pinion.
Ng = No. of teeth in gear.
N.R. = No. of teeth in rack.
L = Length of rack.
P.D. = Pitch diameter.
C.D. = Center distance.
C.P. = Circular pitch.
Wh.D. = Whole depth.
Wg.D. = Working depth.
Add. = Addendum.
Ded. = Dedendum.
C = Clearance.
Th. = Thickness of tooth.
R.D. = Root diameter.

Formulas

$P = \pi \div \text{C.P.}$ or $N \div \text{P.D.}$
$\text{O.D.} = (N + 2) \div P$ or $(N + 2) \times \text{C.P.} \div \pi$ or $\text{P.D.} + 2$ add.
$\text{C.P.} = \pi \div P$ or $\text{P.D.} \times \pi \div N$.
$\text{P.D.} = N \div P$ or $N \times \text{C.P.} \div \pi$ or $\text{O.D.} - 2$ add.
$\text{C.D.} = (Ng + Np) \div 2P$ or $(Ng + Np) \times \text{C.P.} \div 6.2832$.
$\text{Clear.} = 0.157 \div P$ or $\text{C.P.} \div 20$.
$\text{Add.} = 1 \div P$ or $\text{C.P.} \div \pi$ or $\text{C.P.} \times 0.318$.
$\text{Ded.} = 1 \div P$ or $\text{C.P.} \div \pi$ or $\text{C.P.} \times 0.318$.
$\text{Wh.D.} = 2.157 \div P$ or $0.6866 \times \text{C.P.}$
$\text{Th.} = 1.5708 \div P$ or $\text{C.P.} \div 2$.
$N = P \times \text{P.D.}$ or $\pi \times \text{P.D.} \div \text{C.P.}$
$L = \pi \times \text{N.R.} \div P$ or $N \times \text{C.P.}$
$\text{R.D.} = \text{O.D.} - 2\,\text{Wh.D.}$ or $\text{P.D.} - 2\,(\text{Ded.} + C)$.

EXERCISES

1. Determine the diametral pitch of a gear whose circular pitch is 0.5236.

2. Find addendum, dedendum and clearance of a 4 pitch gear.

3. What is the length of add. if P.D. of gear is 2″ and the gear has 20 teeth?

4. Estimate clearance on a 6 pitch gear.

5. Determine center distance of a 25 tooth and a 60 tooth gear, of 10 pitch.

6. What is center distance of two gears of 40 and 60 teeth, 10 P.

7. Find outside diameter of a gear of 30 teeth, pitch diameter 5″.

8. Find outside diameter of a gear of 40 teeth and circular pitch 0.3141.

9. How many teeth are there in a gear of 4 pitch, 8″ pitch diameter?

10. Estimate O.D. of a gear whose circular pitch is 0.500″ and $N = 60$.

11. Find the length of a rack with 30 teeth, of 6 pitch.

12. Find the length of a rack with 60 teeth, of 10 pitch.

13. Find addendum, dedendum and clearance of a 7 pitch gear.

14. What is the P.D. of a gear 4″ O.D. and 8 pitch.

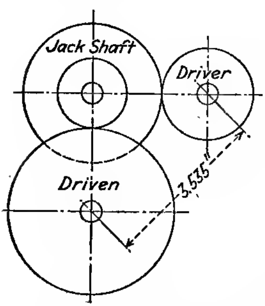
15. Find thickness of tooth of a 6 pitch gear.

16. What is the O.D. of a 40 tooth gear, 4″ pitch diameter?

17. Find the clearance of a 10 pitch gear.

18. Find the pitch diameter of a 30 tooth gear, of 6 pitch.

19. Give all measurements accurate and complete for a toolmaker who wishes to make a master gear of 10 pitch, pitch diameter 5″.



20. Given center distance 5″, ratio 2 to 3, pitch 10; find pitch diameter, outside diameter and number of teeth in each gear.

21. Given approximate center distance $5\frac{1}{8}$″, ratio 15 to 26, 8 pitch; find pitch diameter, outside diameter and number of teeth in each gear.

22. If two parallel shafts (as per sketch) with a center distance of 3.535″ are connected with a jack shaft at right angles and parallel to the others, the jack shaft having two gears on it with a ratio of 2 to 1 and the driving shaft having a 20 T, 10 P gear, what will be the P.D. and O.D. of the gears on the driving shaft, jack shaft and

driven shaft, providing the ratio between the driving shaft and driven shaft is $3\frac{1}{2}$ to 1 and the jack shaft is equal distance from the driving shaft and driven shaft?

23. What number of cutter should be used for cutting a 24 tooth gear; a 40 tooth gear?

Bevel Gearing

Bevel Gears are used to transmit positive rotary motion to shafts at an angle to each other, and in the same plane.

The teeth of a bevel gear are made on a frustum of a cone whose apex is the same point as the intersection of the axes of the shafts.

Bevel gears usually connect shafts running at right angles.

When the angle of the shafts is 90 deg. and the velocity ratio is 1 to 1, then both gears are of the same size, and are

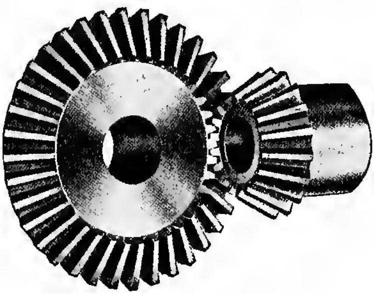


FIG. 1

called **Miter Gears.** If the velocity ratio between two gears is other than 1 to 1, the smaller gear is called the **Pinion.**

When the pitch of two gears is the same, they will mesh properly regardless of number of teeth, providing they have twelve or more teeth.

A gear with less than 12 teeth must be cut special to avoid interference of teeth while rolling.

The P, O.D. and P.D. of a bevel gear are always reckoned on large end of tooth.

Formulas

Tang. of P.C. ang. of pinion	$= Np \div Ng$.
Tang. of P.C. ang. of gear	$= Ng \div Np$.
Pitch diameter	$= N \div P$.
Addendum	$= 1 \div P$ or C.P. $\times$ 0.318 or C.P. $\div \pi$.
Dedendum	$= 1 \div P$ or C.P. $\times$ 0.318 or C.P. $\div \pi$.
Whole depth of tooth	$= 2.157 \div P$ or C.P. $\times$ 0.687.
Pitch cone radius	= P.D. $\div$ (2 $\times$ sin P.C. ang.).
Thickness of tooth	$= 1.571 \div P$ or C.P. $\div$ 2.
Small addendum	= (P.C.R. $-$ B) $\div$ P.C.R. $\times$ add.
Small thickness of tooth	= (P.C.R. $-$ B) $\div$ P.C.R. $\times$ thick.
Angle of add.	= Add. $\div$ P.C.R. = tang.
Angle of ded.	= Ded. $\div$ P.C.R. = tang.
Face angle	= 90 deg. $-$ (P.C. ang. + add. ang.).
Cutting angle	= P.C. ang. $-$ ded. ang.
Angular addendum	= Cos. of P.C. ang. $\times$ add.
Outside diameter	= Ang. add. $\times$ 2 + P.D.
No. of teeth for which to select cutter.	$= \dfrac{N}{\text{Cos. of P.C. Ang.}}$.

P.C. rad.	= Pitch cone radius	$= A$
W. of F.	= Width of face	$= B$
Ang. add.	= Angular addendum	$= C$
Add. ang.	= Addendum angle	$= D$
Ded. ang.	= Dedendum angle	$= E$
P. line	= Pitch line	$= F$
P.C. ang.	= Pitch cone angle	$= G$
Cut. ang.	= Cutting angle	$= H$
O.D.	= Outside diameter	$= I$

P.D.	= Pitch diameter	=	*J*
P.C. ang. G.	= Pitch cone angle of gear	=	*K*
P.C. ang. P.	= Pitch cone angle of pinion	=	*L*
Wh. D.	= Whole depth	=	*M*
Add.	= Addendum	=	*N*
Ded.	= Dedendum	=	*O*
E. ang.	= Edge angle	=	*P*
F. ang.	= Face angle	=	*Q*
Ng	= No. of teeth in gear		
Np	= No. of teeth in pinion		
N	= No. of teeth		
P	= Diametral pitch or pitch		
T	= Thickness of tooth		
N′	= No. of teeth for which to select cutter		

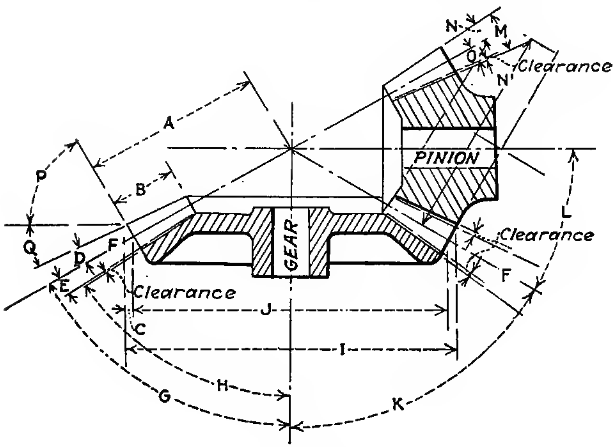


FIG. 11

EXERCISES

(In figuring these problems, make the width of face equal to $\frac{1}{3}$ of the pitch cone radius)

1. Find P.C. rad. of a bevel gear whose P.D. is 4″, P.C. ang. 60 deg. 10 pitch and 40 teeth.

2. Estimate cutting angle of the above gear.
3. Estimate width of face of the above gear.
4. Find addendum angle of the above gear.
5. Find dedendum angle of the above gear.
6. Find outside diameter of the above gear.
7. Find pitch diameter of the above gear.
8. Find whole depth of tooth for a 10 pitch gear.
9. Determine the thickness of cutter at pitch line to use for the first cut on a bevel gear of 30 teeth, 6 pitch and 60 deg. P.C. ang.
10. At what angle from horizontal would you set the dividing head to cut a bevel gear, P.C. ang. 48 deg., 6 pitch and 30 teeth.
11. Estimate face angle of a bevel gear with 30 teeth, 6 pitch and a P.C. ang. of 60 deg.
12. Determine the whole depth of tooth at small end of a bevel gear with 30 teeth, 6 pitch and P.C. ang. of 54 deg.
13. Find the P.D. at the small end of a 40 tooth gear, 8 pitch and a P.C. ang. of 42 deg.
14. Find the outside diameter of a bevel gear with 42 teeth, 6 pitch and a P.C. ang. of 38 deg.
15. Give measurements accurate and complete for a master bevel gear of 50 teeth, 10 pitch and 60 deg. P.C. ang.
16. If two 6 pitch bevel gears, shafts at 90 deg., have a velocity ratio of 3 to 5, with 18 teeth in the pinion, what are the O.D. and face angles of the blanks?
17. In a pair of 2 pitch bevel gears, with shafts at 90 deg., having a velocity ratio of $2\frac{1}{2}$ to 1, the pinion has 24 teeth. Find the face angle, pitch cone angle and cutting angle of both gears.
18. Determine the number of cutter required to cut the bevel gear in problem No. 9.

Worm Gearing

Worm Gearing is used to transmit power between two shafts at 90° to each other, but not in the same plane, and is generally used when it is desired to obtain smoothness of action and great speed reduction from one shaft to another.

The greatest objection to worm gear drives is the excessive sliding friction between the teeth, thus making them very inefficient and subject to heating.

A **Worm** is a screw so cut as to mesh properly with the

teeth of a worm wheel, the included angle of the sides being 29 deg.

The **Worm Wheel** is similar to a spiral spur gear. It usually has a concave face and the tooth spaces are concave and at an angle other than 90 deg. to the side of the gear. These teeth are generally cut by first being indexed and gashed, and afterwards cut to true form with a hob. But if possible they should be cut on a gear hobbing machine, for accurate results.

FIG. 1

A **Hob** is a cutter slightly larger in diameter than the worm, and it appears somewhat like a worm with the exception that flutes are cut into it to form the cutting teeth.

The **Linear Pitch** is the distance from the center of one tooth to the center of the next, measured on the pitch circle.

The **Lead** sometimes differs from the pitch and it is the distance a tooth on the worm would advance in one revolution, or the distance the worm wheel advances in one complete turn of the worm.

Formulas for the Worm

Lead = linear pitch $\times$ no. of separate threads on the worm.
Linear pitch = lead $\div$ no. of separate threads on the worm.
Addendum = linear pitch $\times$ 0.3183.
Whole depth of thread = linear pitch $\times$ 0.6866.
Width of threading tool at end or width of bottom of space = linear P. $\times$ 0.31.
O.D. = P.D. + (2 $\times$ add.).
P.D. = O.D. − (2 $\times$ add.).

P.D. = (2 × center distance) − P.D. of gear.
Root diameter = O.D. − (2 × whole depth of tooth).
Co-tangent of angle of worm tooth or gashing angle of wheel = (P.D. × π) ÷ lead.

Formulas for Worm Wheel

P.D. = (no. of teeth in gear × linear pitch of worm) ÷ π.
Throat diameter = P.D. of worm wheel + 2 × add.
Radius of throat = $\frac{1}{2}$ of O.D. of worm − (2 × add. of worm).
Center distance = (P.D. of worm + P.D. of gear) ÷ 2.
O.D. = (throat radius − throat radius × cosine of $\frac{1}{2}$ face angle) × 2 + throat diameter of wheel.

Worm

A = Clearance.
B = Working depth of tooth.
C = Whole depth of tooth.
D = O.D. of worm.
E = P.D. of worm.
F = Ang. of helix.
G = Linear pitch.

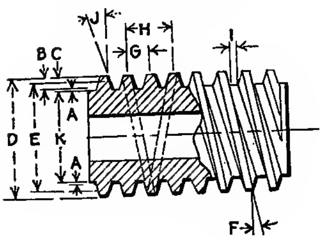


FIG. II. WORM

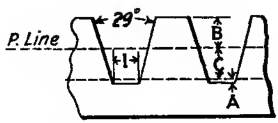


FIG. III. WORM THREAD (enlarged view)

A = Clearance.
B = Addendum.
C = Dedendum.

H = Lead.
I = Thickness of end of tool or bottom of space.
J = $\frac{1}{2}$ ang. of tooth.
K = Root diameter of worm.

Worm Wheel

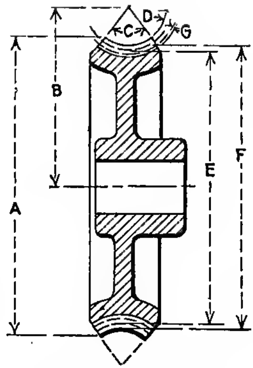


FIG. IV. WORM WHEEL

A = O.D. of worm wheel.
B = Center distance of worm and worm wheel.
C = Ang. of face.
D = Throat radius.
E = Pitch diameter.
F = Throat diameter.
G = Clearance.

EXERCISES

1. Determine pitch diameter of worm wheel—number of teeth in wheel 30 and linear pitch 0.200″.

2. Find throat diameter of a worm wheel of 40 teeth, linear pitch of worm 0.230″.

3. Find outside diameter of a worm wheel whose face angle is 70 degrees, throat radius is $\frac{1}{2}$″, number of teeth 32, and linear pitch of worm 0.200″.

4. Find center distance of a worm gear of 48 teeth, linear pitch of worm 0.200″, outside diameter of worm $1\frac{1}{4}$″.

5. Find angle of worm tooth or gashing angle of wheel, if the outside diam. of the worm is $1\frac{1}{2}$″, linear pitch 0.240″, with a double thread.

6. Find root diameter of a worm whose outside diameter is $1\frac{1}{8}$″ and linear pitch 0.175″.

7. Determine thickness of thread tool at small end for a worm whose linear pitch is 0.173″.

8. Estimate velocity ratio of a worm and worm gear if the gear has 30 teeth. (Worm has a triple thread.)

9. Find radius of curvature of the worm wheel throat, if the pitch of the worm is 0.150″ and the outside diameter is 1″.

10. Determine all measurements accurate and complete for a mechanic to make a set of master worm gears; linear pitch of worm 0.200″, double thread, outside diameter of worm $1\frac{1}{2}$″, number of teeth in wheel 32, and a face angle of 70 deg.

Spiral Gearing

Spiral or **Helical** gears are used to drive shafts at angles to each other but not in the same plane.

The **Velocity Ratio** depends upon the number of teeth and their helical angles.

The velocity ratio of two spiral gears is proportional to their pitch diameters only when the spiral angles of the gears are the same; or 45 deg. when the axes are at 90 deg. to each other.

The sum of the **Spiral Angles** of two mating gears must always equal the angle between the shafts.

The **Normal Pitch** is the pitch measured at right angles with the tooth.

The **Tooth Angle** is the angle the tooth makes with the axis of the gear.

The **Center Angle** of a pair of spiral gears is the angle made by the axes of the gears.

The **Normal Diametral Pitch** is the D.P. of the cutter used for cutting the teeth in the spiral gear.

The **Thickness of Cutter** at the pitch line for milling spiral gears should equal $\frac{1}{2}$ of the normal circular pitch.

(*Abbreviations Generally Used in Spiral Gearing*)

FIG. 1

a = small gear.
A = large gear.
α = spiral angle.
r = center angle (or ang. between shafts).
Pn = normal diameter pitch (pitch of cutter).
N = no. of teeth.
N' = no. of teeth for which to select cutter.
C = center distance.
L = lead of tooth helix.
Tn = normal thickness of tooth at pitch line.

Add. = addendum.
Ded. = dedendum.
W = whole depth of tooth.
D = pitch diameter.
O = outside diameter.
V = velocity of large gear.
v = velocity of small gear.
T = thickness of cutter.
K = no. of teeth for which to select cutter for large gear.

The **Number of Cutter** to use for cutting spiral gears is not selected with reference to the actual number of teeth in the spiral gear, but of an imaginary spur gear arranged at right angles to the normal pitch.

Formulas

$$r = \alpha_A + \alpha_a.$$

$$D = \frac{N}{Pn \cos \alpha}.$$

$$C = \frac{Da + DA}{2}.$$

$$L = \pi D \times \cot \alpha.$$

$$N' = \frac{N}{(\cos \alpha)^3}.$$

$$\text{Add.} = \frac{1}{Pn}.$$

$$\text{Ded.} = \frac{1}{Pn}.$$

$$W = \frac{2.157}{Pn}.$$

$$Tn = \frac{1.571}{Pn}.$$

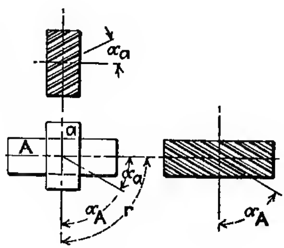


FIG. II

$$O = D + 2B.$$

$$v : V :: DA \times \cos \alpha_A : Da \times \cos \alpha_a.$$

$$K = \frac{N}{(\cos \alpha)^3}.$$

$$\text{Normal pitch} = \frac{P}{\text{cos of tooth ang.}}.$$

EXERCISES

1. Two 25 tooth spiral gears, with a velocity ratio of 1 to 1 and shafts at 90 deg. are to be cut with a 10 P. cutter. Find P.D., O.D., whole depth, add, center distance and lead of tooth helix.

2. Two spiral gears with axes at 90 degrees, velocity ratio 2 to 3, are to be cut. The approximate P.D. of small gear is $2\frac{3}{4}''$, and its tooth angle is 30 deg., and normal pitch is 10. Find the pitch diam., no. of teeth, O.D., and no. of cutters for both gears, and also center distance.

3. Find the O.D. of two spiral gears that have 18 teeth, with a velocity ratio of 1 to 1, cut with an 8 pitch cutter, shafts at 90 deg.

4. What will be the center distance between 2 spiral gears with a velocity ratio of 1 to 1, shafts at 90 deg. each having 80 teeth, if the normal pitch is 12.

5. Two spiral gears are required to have a velocity ratio of 1 to 1, shafts at 90 deg. to each other, a 6 pitch cutter to be used with an O.D. to be as close as possible to $5''$. Find P.D., number of teeth, normal cir. pitch, center distance and number of cutters to cut the above gears.

6. Two spiral gears with axes at 90 deg. to each other, velocity ratio 2 to 1, normal pitch 12, center distance $2\frac{1}{4}''$. number of teeth in small gear 12, tooth angle 38 deg. 20 min. Find P.D., O.D., cir. pitch, normal pitch, and number of cutter for both gears, also number of teeth and tooth angle of large gear.

7. What is the velocity ratio of 2 spiral gears if the large gear has a P.D. of $5''$ and tooth angle of 60 deg. and the small gear has an approximate P.D. of $2\frac{1}{2}''$ with a tooth angle of 30 deg.

REVIEW EXERCISES

1. A pole was 2/7 under water, the water rose 8 ft.; and then there was as much under water as had been above water before. How long is the pole?

2. A boat goes $16\frac{1}{4}$ miles per hour down stream and 10 miles an hour up stream. If it is $22\frac{1}{2}$ hours longer in coming up stream than going down, how far down did it go?

3. Land worth $1000 an acre is worth how much a front foot of 90 ft. depth, counting off 1/10 for streets?

4. A machinist took a cut on a lathe off a piece of round cast iron 2″ deep; the piece was 8″ in diameter and 6″ long. How much did the material weigh that was cut away?

5. Find the length of a minute hand on a clock whose extreme point moves 4″ in 3 min. 28 sec.

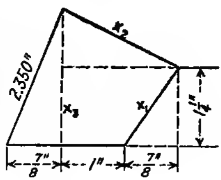
6. How many 1″ balls can be put in a box which measures inside 5″ × 10″ × 10″ deep?

7. A 12″ ball is in the corner where walls and floor are at right angles. What must be the diameter of a ball placed in the back of this, and which will touch the 12″ ball and also the same floor and walls?

8. If $\frac{1}{4}$″ of rain fell, how many barrels of water will be caught by a cistern which drains a flat roof 52 ft. by 38 ft.?

9. What peripheral distance would a lathe tool travel on threading a piece of work 2″ in diameter and 10″ long, if the lead is $\frac{1}{8}$″?

10. Four holes are required in a jig as per sketch below. Find dimensions (x_1, x_2 and x_3).



11. If 15 men cut 480 cords of wood in 10 days of 8 hours each, how many boys will it take to cut 1152 cords of wood only 2/5 as hard, in 16 days of 6 hours each, provided that while working, a boy can do only $\frac{3}{4}$ as much as a man, and that $\frac{1}{3}$ of the boys are idle at a time throughout the work?

12. Extract the cube root of 14172488.

13. Extract the cube root of 2.6 (to three decimals).

14. What would a taper piece of cast iron weigh, which was 10″ long, $\frac{3}{4}$″ in diameter at the small end and 3″ in diameter at the large end?

15. How many pounds of round cast iron stock would you order to make the above piece without allowing for waste on ends and on large diameter.

PART III

Dovetail Slides

In the shop dovetail slides are usually measured by placing round plugs or rods of such diameter that they will bear on the angular surface and then measuring overall or the distance between the plugs or rods.

Dovetails are generally dimensioned as shown in Fig. I, and the above method is used for checking the dimensions as in the shop it is impossible to measure accurately the overall dimensions from edge to edge due to these edges or measuring surfaces not being absolutely sharp.

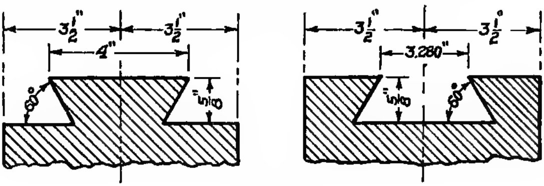


FIG. I

To obtain the dimensions X and Y (Fig. II) which are

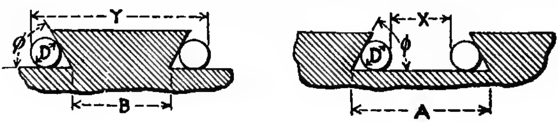


FIG. II

necessary for the practical measuring of dovetail slides, the following formulas may be used:

$$X = A - [D(1 + \cot \tfrac{1}{2}\phi)],$$
$$Y = D(1 + \cot \tfrac{1}{2}\phi) + B.$$

Example: Find the distance Y on a dovetail slide if the

blue print gives the dimension A as 3″, B as 1.846″ and the angle 60°, providing plugs 0.750″ in diameter are used.

Formula: $Y = D(1 + \cot \frac{1}{2}\phi) + B = 0.750(1 + 1.732) + 1.846 = 0.750 \times 2.732 + 1.846 = 2.049 + 1.846 = 3.895''$ (Ans.).

Example: Find the distance X in the above problem.

Formula: $X = A - [D(1 + \cot \frac{1}{2}\phi)] = 3 - [0.750(1 + 1.732)] = 3 - [0.750 \times 2.732] = 3 - 2.049 = 0.951''$ (Ans.).

EXERCISES

1. What will be the overall length in measuring a male dovetail, if the following data is given on the blue print: angle 70°, width at bottom 2.886″, providing plugs $\frac{3}{4}''$ in diameter are used.

2. What will be the distance between to $\frac{5}{8}''$ plugs placed in a female dovetail which measures 2″ wide at the bottom and the included angle is 60°?

3. What will be the length Y in Fig. II, if the angle is 50 degrees, $D = 0.750''$ and the width B is 6″?

4. Find the distance X in Fig. II? If D is $\frac{1}{4}''$, $\phi = 60°$, the depth equals one inch and the width A is 2.640″.

5. What should be the distance Y of a properly machined dovetail to fit the dovetail in the above problem, if a clearance of 0.002″ is allowed for a sliding fit, providing $\frac{1}{4}''$ plugs are used?

Screw Threads

There are several different forms of threads such as the United States Standard, Sharp V, Square, Whitworth, Acme, Briggs pipe, etc.

The most commonly used threads are the United States standard, sharp V, and square threads.

Standard Sharp V Thread (Fig. I).—The sides of the thread form an angle of 60 degrees with each other, and is theoretically sharp at the top and bottom.

$$p = \text{pitch} = \frac{1}{\text{no. threads per inch}}.$$

d = depth = $p \times \cos 30$ deg. or $0.866\,p$ or $\dfrac{0.866}{\text{no. threads per inch}}$.

D = outside diameter (Fig. II).

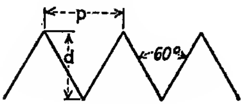


FIG. I

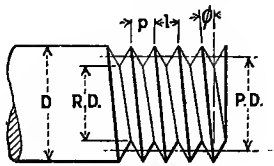


FIG. II

P.D. = pitch diameter = $D - d$ or $D - \dfrac{0.866}{N}$.

R.D. = root diameter = $D - (2d)$ or $D - \left(\dfrac{0.866}{N} \times 2\right)$.

l = lead = distance the screw advances if turned around one complete revolution. In a single threaded screw the pitch and the lead are equal. In a double threaded screw the lead is equal to twice the pitch, etc. (Figs. III and IV).

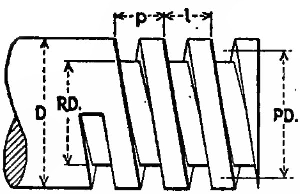


FIG. III. SINGLE SQUARE THREAD

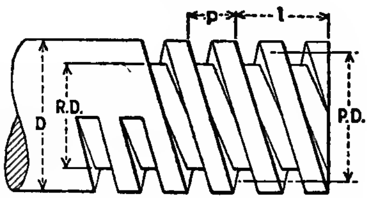


FIG. IV. DOUBLE SQUARE THREAD

United States Standard Thread (Fig. V).—The sides of this thread also form an angle of 60 degrees with each other, but the thread is flattened at the top and bottom and this flat is equal to $\frac{1}{8}$ of the pitch.

$$p = \text{pitch} = \frac{1}{\text{no. threads per inch}}.$$

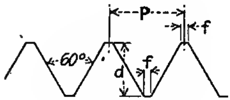


Fig. V

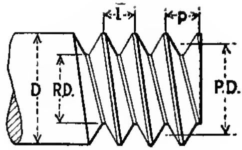


Fig. VI

$$d = \text{depth} = \tfrac{3}{4} \times p \times \cos 30 \text{ deg., or } 0.6495 \times p \text{ or } \frac{0.6495}{\text{no. threads per inch}}.$$

D = outside diameter (Fig. VI).

$$\text{P.D.} = \text{pitch diameter} = D - d \text{ or } D - \frac{0.6495}{N}.$$

$$\text{R.D.} = \text{root diameter} = D - (2d) \text{ or } D - \left(\frac{0.6495}{N} \times 2\right).$$

$$f = \text{flat} = \text{width of flat at top and bottom} = \frac{P}{8} \text{ or } \frac{1}{8 \times \text{no. threads per inch}}.$$

The pitch diameter is generally measured directly with a thread micrometer, but a method for measuring accurately the pitch diameter by means of an ordinary micrometer and three wires of equal diameter, can be used. The three wires are arranged as shown in Fig. VII, one wire being placed in the angle of thread on one side of the

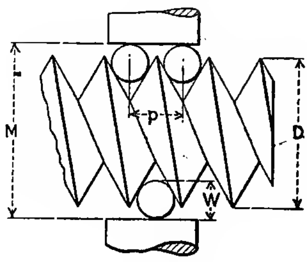


Fig. VII

screw, and the other two on the opposite side, then measuring over the whole with a micrometer. The reading will be according to the following formulas, if the pitch diameter is correct.

For sharp V thread.	For U.S.S. thread.
$D = M - 3W + 1.732p.$	$D = M - 3W + 1.5155p.$
$M = D - 1.732p + 3W.$	$M = D - 1.5155p + 3W.$

The **Tap Drill** size theoretically is equal to the root diameter of the given tap, but in practice it is always a little larger, to prevent excessive strain on the tap, thus increasing production. (See appendix, table VIII.)

The Briggs standard pipe thread is made with an angle of 60 degrees. It is slightly rounded at the top and bottom. The taper of the thread on the diameter equals 1/16″ per inch or 3/4″ per foot. For number of threads per inch, tap drill sizes, etc. (See appendix, table IX.)

Square Thread (Fig. VIII).—The sides of the square thread are parallel and the depth of the thread is equal to the width of space between the teeth. This space is theoretically equal to one-half of the pitch. It is necessary in practice to make the space in the nut a trifle wider than the thread so as to have a running fit between the screw and nut.

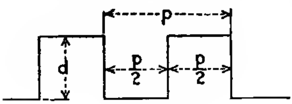


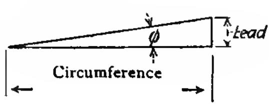
FIG. VIII

The **Screw Thread Angle** (angle of helix) (ϕ) Fig. II varies with the P.D. and the lead of the screw.

The following formula may be used:

$$\frac{\text{lead}}{\text{P.D.} \times \pi} = \text{tang. of helix angle.}$$

The following graphical method may be used:



EXERCISES

1. Find the depth of V and U.S.S. threads of $\frac{1}{8}''$ pitch.
2. Find the depth of V and U.S.S. threads of 1/12″ pitch.
3. What is the R.D. of a $\frac{3}{4}'' \times 10$ U.S.S. thread?
4. What is the P.D. of a $1'' \times 8$ V thread?
5. What is the theoretically correct tap drill size for a $\frac{5}{8}'' \times 11$ V thread?
6. What is the theoretically correct tap drill size for a $\frac{7}{8}'' \times 9$ U.S.S. thread?
7. What is the theoretically correct tap drill size for a $\frac{3}{8}'' \times 16$ V thread?
8. What must be the width of flat on a U.S.S. thread having 7 threads per inch? Also one having $2\frac{1}{4}$ threads per inch?
9. Find the width of tool and boring size of a 1″, $\frac{1}{4}''$ pitch, square thread.
10. What multiple thread must be cut on a screw so that it will advance $\frac{1}{2}''$ in two-thirds of a revolution, providing the screw has a $\frac{1}{4}''$ pitch, square thread. Also figure the distance the tool must be fed into the work.
11. Give all dimensions necessary for making a master plug gage $1\frac{3}{8}'' \times 6$ standard sharp V thread.
12. What will be the micrometer reading of a $\frac{1}{2}'' \times 12$ U.S.S. thread if the three wire system is used, providing the wires used are 0.070″ in diameter?
13. What will be the correct micrometer reading of a $2\frac{3}{4}'' \times 10$ V thread if the three wire system is used, and the wires are 5/64″ in diameter?
14. What will be the correct micrometer reading of a $2\frac{5}{16}'' \times 16$ U.S.S. thread if the three wire system is used, wire being 0.075″ in diameter. Also find the P.D. and R.D.
15. Find the helix angle of a $\frac{1}{8}''$ P. screw, 1″ P.D.

Lathe Change Gears

In cutting threads on a lathe, the lead screw must be taken as the first factor and the main spindle as the second. If the lead screw has 5 threads per inch, and the lead screw makes five complete revolutions, the carriage will travel one inch, or the threading tool will have traveled the same distance along the piece to be threaded. If the spindle and

lead screw are geared one to one, the spindle will make the same number of revolutions as the lead screw. Therefore the same lead will be cut on the work as on the lead screw.

If the gear on the spindle is $\frac{1}{2}$ the size as that on the lead screw, the spindle will make twice the number of revolutions as the feed screw, the spindle revolving 10 times while the tool moves one inch. Therefore 10 threads will be cut.

If the gear on the lead screw and main spindle are connected with an idler gear (which does not change the ratio) this is called **Simple Gearing** (Fig. I).

Sometimes it is not possible to obtain the correct ratio with two gears, then two more gears must be put into the gear train; this is called **Compound Gearing** (Fig. II).

To find the gear ratio between spindle and lead screw in simple gearing, the following formula is used:

$$\frac{\text{Threads per in. of lead screw}}{\text{Threads per in. to be cut}} = \frac{\text{teeth in gear on spindle stud}}{\text{teeth in gear on lead screw}}.$$

To cut 12 threads per inch with a lathe that has a 6 pitch lead screw according to the above formula.

$$\frac{6}{12} = \frac{1}{2} \text{ or } \frac{30}{60} \begin{array}{l} = \text{teeth in gear on spindle.} \\ = \text{teeth in gear on lead screw.} \end{array}$$

For compound gearing the same formula as above is used, except that we divide both numerator and denominator into two factors. Thus:

$$\frac{6}{12} = \frac{2 \times 3}{4 \times 3} = \frac{30 \times 40 \text{ drivers.}}{60 \times 40 \text{ driven.}}$$

The following are the standard gears put out with a Reed lathe having a 5 P lead screw: 25–30–35–40–40–45–50–55–60–65–69–70–75–80–90.

The following are the standard gears put out with the Pratt & Whitney lathe, having a 6 P lead screw: 30–40–50–60–65–70–75–80–90–95–100–105–110–115–120.

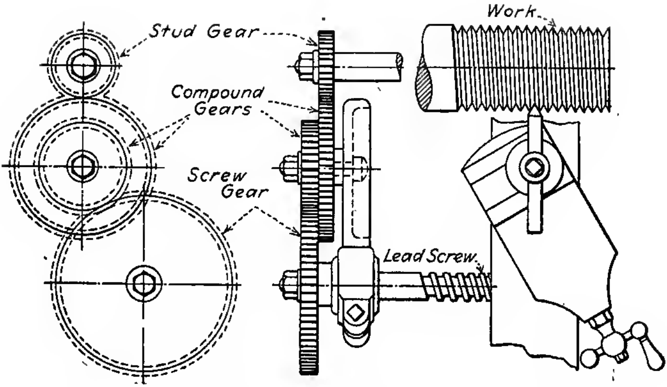


FIG. I. SIMPLE TRAIN OF GEARS FOR THREAD CUTTING

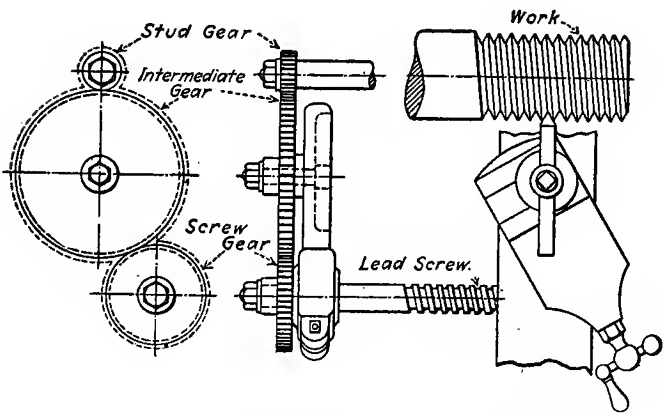


FIG. II. COMPOUND GEARS FOR THREAD CUTTING

EXERCISES

1. What change gears can be used to cut a 13 P thread with a lathe that has a 4 P, lead screw, using a stud gear of 20 teeth?

2. What change gears can be used to cut a $11\frac{1}{2}$ P thread when lead screw is 5 P, using a 30 tooth gear on stud.

3. Find the stud gear to be used to cut 18 threads per inch, when lead screw is 6 P, and screw gear has 90 teeth.

4. What pitch thread can be cut with a 6 P lead screw if the drivers have 75 and 80 teeth and the driven gears have 50 and 110 teeth?

5. What screw gear will be used to cut a 24 P double-thread screw when lead screw is 6 P and the stud gear has 60 teeth?

6. What change gears can be used to cut a 5 P thread when the lead screw on the lathe is 6 P?

7. What change gears can be used to cut a 9 P thread when lead screw is 6 P, and the smaller gear in the set has 30 teeth?

8. The spindle gear in a compound gear train has 25 teeth. On the idler stud are two gears, driven 60 teeth and driver 30 teeth, and the lead screw has 80 teeth. How many turns does the spindle make for one turn of the lead screw?

9. Using an 80 tooth gear on the lead screw, and a 25 tooth gear on the stud, how many threads can be cut per inch if the lead screw has 8 threads per inch.

10. Using a 110 gear on the lead screw and a 75 on the stud, with compound driven and driver gears of 50 and 80 teeth respectively, how many threads per inch will be cut if the lead screw is 6 P?

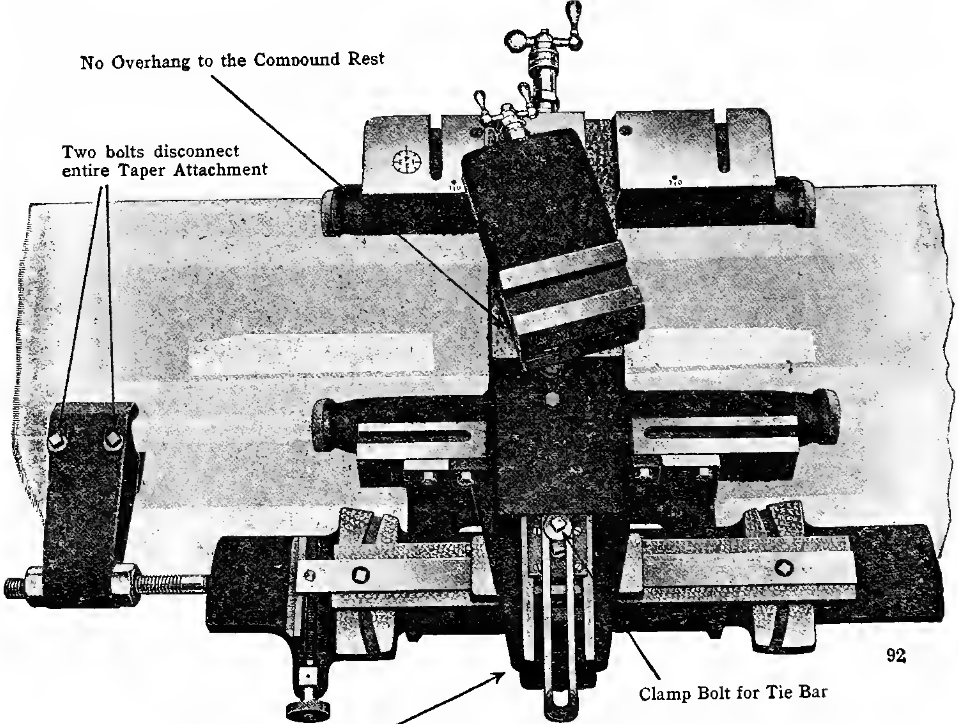
Indexing

The **Dividing Head** is used to divide the periphery of a piece of work into any desired number of spaces or divisions.

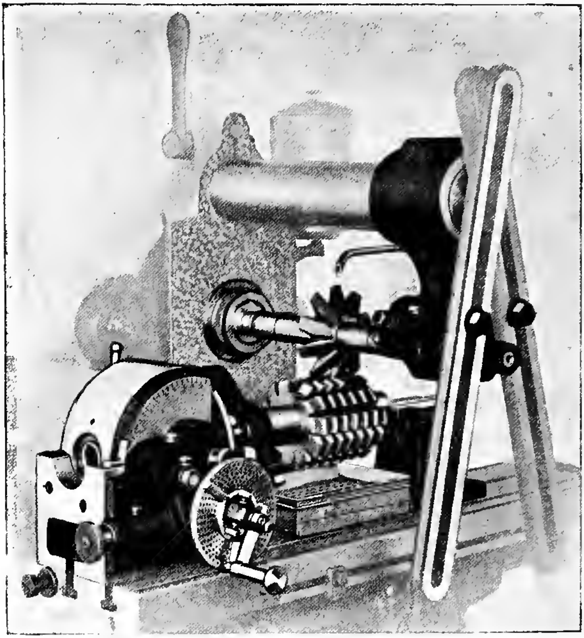
There are generally three interchangeable plates with each dividing head. The following list gives the usual number of holes per circle on the three plates.

Plate	Number of Holes in the Various Circles
1	15–16–17–18–19–20
2	21–23–27–29–31–33
3	37–39–41–43–47–49

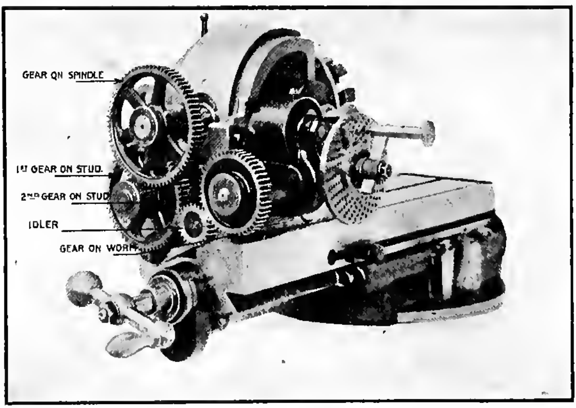
Some dividing heads have only one plate, in this case the



LATHE CARRIAGE SHOWING TAPER ATTACHMENT



Simple Indexing



Differential Indexing

plate has holes on each side as follows: One side—24–25–28–30–34–37–38–39–41–42–43; and on the other side—46–47–49–51–53–54–57–58–59–62–66.

On all standard dividing heads it requires 40 turns of the index crank to revolve the dividing head spindle once.

Simple Indexing

To find the **Number of Turns** of the index crank for any number of divisions necessary on the work, divide the number of turns required for one revolution of the dividing head spindle (40) by the number of divisions wanted.

N = Number of divisions required.

R = Number of turns of the crank for a given division.

$$R = \frac{40}{N}.$$

Example: Find the indexing required for 50 divisions.

Solution: $R = 40/N$ or $40/50$ or $4/5$ of a revolution. Any plate divisible by 5 may be used. In this case, taking the 20 hole circle. Then $4/5$ of 20, or 16 will equal the number of holes to be moved in the 20 hole circle, for each division.

Compound Indexing

Compound Indexing is sometimes used to obtain divisions which cannot be secured by simple indexing. In this method the crank is first turned a certain amount in the regular way, and then the index plate is also turned either in the same or opposite direction in order to locate the index crank in the proper position. The back locating pin not being adjustable, is in line only with the outside circle of holes in the index plates. Thus, the circles with the 20, 33 and 49 hole circles must be used for compounding.

This method was used extensively in the past, but the chances of errors are too great in making the complicated

indexing moves, and even if properly operated, the spacing is more or less inaccurate. Therefore this method is very seldom used at present, and has been largely superseded by differential indexing.

To find what circles of holes can be used in compound indexing, the following method must be used:

(*a*) Set down the number of divisions required and resolve into factors.

(*b*) Choose at random two circles of holes, subtract one from the other and factor the difference.

(*c*) Place the factors (*a*) and (*b*) above a horizontal line.

(*d*) Factor the number of turns of index crank required for one revolution of the spindle (40).

(*e*) Factor the number of holes in each of the chosen circles (*b*).

(*f*) Place the factors obtained in (*d*) and (*e*) below the horizontal line.

If all the factors above the line can be cancelled by those below the line, the two circles chosen will give the required number of divisions chosen. If not, other circles must be chosen and another trial made.

Example: If 50 divisions are required and that circles 20 and 16 are to be chosen for trial divisors.

$a = 50 = 5 \times 5 \times 2.$

$b = 20 - 16 = (4) = 2 \times 2.$

$c = 5 \times 5 \times 2 \times 2 \times 2.$

$d = 40 = 2 \times 2 \times 2 \times 5.$

$e = (20) = 5 \times 2 \times 2. \qquad (16) = 2 \times 2 \times 2 \times 2.$

$f = 2 \times 2 \times 2 \times 5 \times 5 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2.$

$$\frac{5 \times 5 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 5 \times 5 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2} = \frac{1}{64} \text{ (Ans.)}$$

Thus, the product of the remaining factors below the line = 64. This means that we can index 1/50 of a revolution

by turning the crank forward 64 holes in the 20 hole circle and the index plate backward 64 holes in the 16 hole circle. This movement may also be reversed, 64 holes in the 16 hole circle and the index plate backward 64 holes in the 20 hole circle, without affecting the result.

Differential Indexing

Differential Indexing is on the same principle as compound indexing except that the index plate is revolved by suitable gears which connect it to the spiral head spindle.

The rotary or differential motion of the index plate takes place when the crank is turned, which turns the plate either forward or backward as may be required. The result is that the actual movement of the crank, in indexing, is either more or less than the movement in relation to the index plate.

The differential method cannot be used in connection with spiral milling, because the spiral head spindle is geared to the lead screw of the milling machine.

The amount of rotation of the index plate may be regulated by the difference in velocity ratios of the change gears.

Example: Find the indexing required for 81 divisions.

Solution: By simple indexing the index crank would be rotated through 40/81 of a turn for each division, but as there is no plate with 81 divisions, the spacing is impossible: therefore another fraction is selected whose value is near 40/81, say 40/84 or 10/21, then a 21 hole circle can be used, indexing in this way for 81 divisions, giving 81 × 10/21 = 810/21 or 38 12/21 complete turns of the index crank or 1 9/21 turns less than the 40 required for one complete turn of the work. By using gears in the ratio of 1 9/21 to 1, the index plate will make 1 9/21 revolutions, which with 38 12/21 turns of the crank will make the 41 turns required. Thus the gears will be in the ratio of

$$1\frac{9}{21} \text{ or } \frac{30}{21} = \frac{6 \times 5}{7 \times 3} \text{ or } \frac{48 \times 40 \text{ driven}}{56 \times 24 \text{ drivers}} \text{ (Ans.).}$$

If the motion of the index plate must be in the direction opposite to the movement of the index crank, idler gears must be used.

The following gears are generally available for differential indexing: 24–24–28–32–40–44–48–56–64–72–86–100.

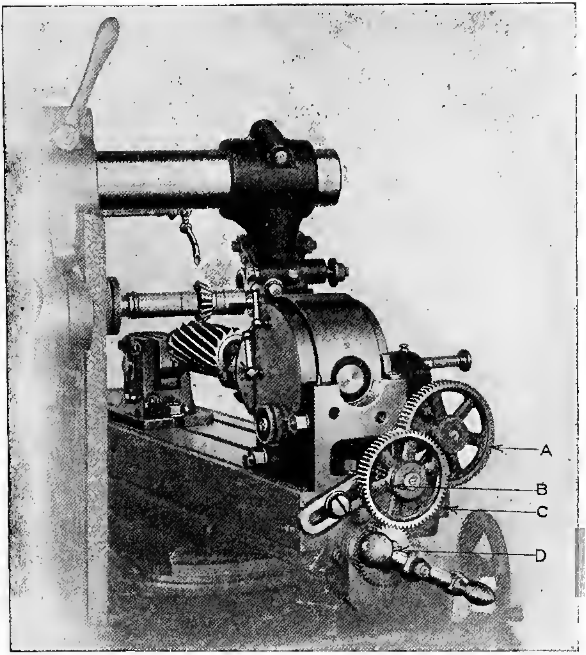
Angular Indexing

With the standard index head, where 40 turns of the index crank are required, for one revolution of the work, one turn of the crank equals 1/40 of 360 deg. or 9 deg.

Thus, if one complete turn of the index crank equals 9 deg., 2 holes in the 18 hole circle or 3 holes in the 27 hole circle must equal 1 deg., or 1 hole in 18 hole circle will equal $\frac{1}{2}$ deg., and 1 hole in the 27 hole circle will equal $\frac{1}{3}$ of a deg.

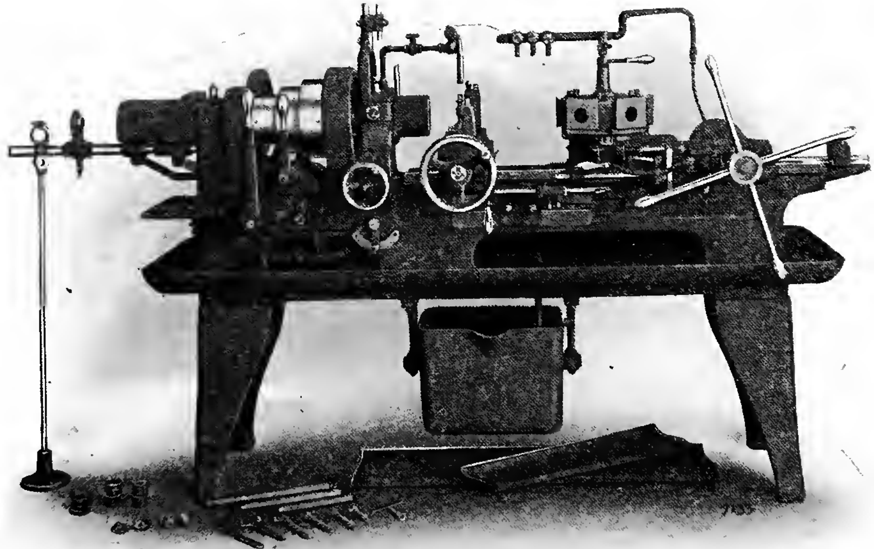
EXERCISES

1. What is the simple indexing for 12 divisions?
2. " " " " " " 28 "
3. " " " " " " 340 "
4. " " " " " " 85 "
5. " " " " " " 115 "
6. " " " compound indexing for 69 divisions?
7. " " " " " " 231 "
8. " " " " " " 87 "
9. " " " " " " 99 "
10. " " " " " " 147 "
11. " " " differential " " 51 "
12. " " " " " " 57 "
13. " " " " " " 71 "
14. " " " " " " 101 "
15. " " " " " " 352 "
16. " " " angular indexing for 13 degrees?
17. " " " " " " $11\frac{1}{3}$ "
18. " " " " " " 5 deg. 20 min.?
19. " " " " " " 21 " 30 "
20. " " " " " " 7 " 40 "



SPIRAL MILLING

A—Gear on worm (driven).
B—First gear on stud (driver).
C—Second gear on stud (driven).
D—Gear on screw (driver).



$1\frac{1}{2} \times 18$-Inch Turret Lathe

Spiral Milling

Spiral Milling is attained by the use of an index head so geared to the longitudinal feed screw of the milling machine, to impart a rotary motion to the work as it is fed along under the cutter by the action of a train of gears.

The **Lead of a Helix** or **Spiral** is the distance, measured along the axis of the work, which the spiral makes in one full turn around the work.

By the **Lead of the Milling Machine** is meant the distance the table will travel while the index head spindle makes one complete revolution when the gear ratio between the feed screw and the worm gear stud is 1 to 1.

Rule.—Lead of milling machine equals the revolutions of the feed screw required for one revolution of the index head spindle with equal gears times the lead of the feed screw.

$$\frac{\text{Lead of spiral}}{\text{Lead of machine}} = \frac{\text{product of driven gears}}{\text{product of driving gears}}.$$

In finding the change gears to be used in a compound train, place the lead to be cut, in the numerator, and the lead of milling machine, in the denominator, then resolve the fraction into its factor and multiply each pair of factors by the same number until suitable number of teeth in change gears are obtained.

The following change gears are available on most milling machines: 24–24–28–32–40–44–48–56–64–72–86–100.

Example: Find required gears to cut a 24″ lead with a 10″ lead milling machine.

$$\frac{24}{10} = \frac{6 \times 4}{2 \times 5} = \frac{(6 \times 12) \times (4 \times 8)}{(2 \times 12) \times (5 \times 8)} = \frac{72 \times 32 = \text{driven gears}}{24 \times 40 = \text{driving gears}}$$

(Ans.).

The **Helix Angle,** or angle to which the table is set in spiral milling, is found by the following formula:

$$\text{tangent of helix angle} = \frac{\pi \times \text{diameter of work}}{\text{lead of work}}.$$

A graphical method of determining the helix angle is shown in Fig. I.

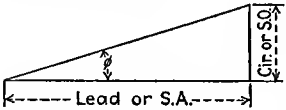


FIG. I

Draw a base line equivalent to the lead and a vertical line equal to the circumference, then by connecting these two lines by a hypotenuse and measuring the angle (ϕ) with a protractor the approximate helix angle may be obtained.

EXERCISES

1. What change gears are required for a spiral index head to cut a 48″ spiral or lead?

2. What change gears are necessary to cut a 40″ lead?

3. What change gears are necessary to cut a $1\frac{1}{2}$″ lead?

4. What lead will the following gears cut: gear on worm 56. 1st gear on stud 28, 2d gear on stud 24, gear on screw 48?

5. What lead or spiral can be cut with the following gears: gear on worm 40, 1st gear on stud 24, 2d gear on stud 24, gear on screw 32?

6. What gears and what angle must the milling machine table be set at to cut a spiral groove, one complete turn on a piece 8″ long and 2″ in diameter?

7. What angle must a milling machine table be set at, if the following gears are used: gear on worm 86, 1st gear on stud 48, 2d gear on stud 56, gear on screw 44, to cut a spiral groove, one complete turn on a piece 2″ in diameter?

8. What gears are required to cut a 60″ lead?

9. What gears are required to cut a 32″ lead?

10. What gears will be necessary to cut a 20 deg. spiral groove one complete turn on a piece 5.460″ in circumference?

Friction

Friction is the resistance to motion which takes place between two bodies at their surface of contact, and depends upon the force with which the bodies are pressed together and upon lubrication. The force of friction will always act

in a direction opposite to that in which the body tends to move.

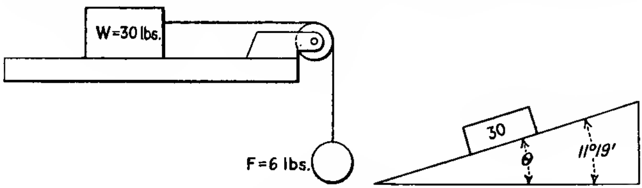
Coefficient of Friction is the ratio of the force required to slide a body along a horizontal plane surface to the weight of the body.

When a body W is just on the point of moving by a force F, the amount of friction between the body and the table is called **Static Friction.**

Example: A body weighing 30 lbs. (W) rests on a horizontal surface. The force required to keep it in motion along the surface is 6 lbs. (F). Find the coefficient of friction.

$$\text{Coefficient of friction } f = \frac{F}{W} = \frac{6}{30} \text{ or } 0.20 \text{ (Ans.).}$$

The coefficient of friction is equal to the tangent of the angle of repose, which is the angle of inclination to the hori-



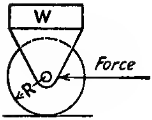
zontal of an inclined plane on which the body will just overcome its tendency to slide. This angle is usually denoted by θ.

$$\text{Coefficient of friction } f = \text{tangent } \theta.$$

$f = 0.20 =$ tang. θ or 11 deg. 19 min., or the angle at which the above weight would repose.

A greater force is required to start a body from a state of rest than to merely keep it in motion, because the friction of rest is greater than the friction of motion.

When a body rolls on a surface, the force resisting the motion is termed **Rolling Friction,** and has a different f value than sliding friction.



Let W = total weight in lbs. of rolling body or load on wheels.

R = radius of wheel, in feet.

f = coefficient of rolling friction.

Then

$$\text{resistance to rolling in lbs.} = \frac{W \times f}{R}$$

when force is applied radially, and parallel to plane.

Table—Value of Coefficient of Friction (f) for Low Pressures (Sliding Friction)

Bronze on bronze (dry	0.20
Bronze on cast iron (dry)	0.21
Cast iron on cast iron (lubricated)	0.15
Cast iron on hard wood (dry)	0.49
Cast iron on hard wood (lubricated)	0.19
Leather on cast iron (dry)	0.56
Hardwood on hard wood (dry)	0.48
Steel on cast iron	0.20
Steel on brass	0.15
Steel on steel	0.14

Rolling Friction

Iron on iron	0.003
Wood on wood	0.005
Iron on wood	0.018
Rubber on asphalt	0.025

EXERCISES

1. What is the f between two bodies, if it requires a 50 lb. force to move a 175 lb. weight?

2. What is the f between cast iron, if it requires a pull of 15 lbs. to slide a lathe carriage which weighs 80 lbs. upon its ways?

3. What is the f of cast iron, if it requires a 26 lb. force to slide a 145 lb. shaper ram upon its ways?

4. What is the angle of repose of bronze on cast iron, if it requires a force of 1 lb. to slide a 48 lb. weight?

5. What is the angle of repose for bronze on bronze?

6. What is the angle of repose for hardwood on hardwood?

7. What is the angle of repose for steel on cast iron?

8. If the angle of repose is 8 deg. 32 min. for cast iron on cast iron, what force will be required to pull a 500 lb. cast iron weight upon a cast iron surface plate?

9. A flywheel weighing 400 lbs. is being rolled into a doorway $3\frac{1}{2}$ ft. above the ground on a plank 16 ft. long. How much power must be applied parallel to the plank to keep the flywheel from rolling back?

10. What force applied parallel with the floor, will be required to roll a 36″ armature weighing 4125 lbs. along a floor, if f is equal to 0.012?

Electricity

A **Volt** is a unit of electrical pressure or potential difference (P.D.) or the electro-motive-force (e.m.f.) required to cause a current of one ampere to flow through a resistance of one ohm.

An **Ampere** is a unit of current strength, or the quantity of flow, or the quantity of current which will flow through a resistance of one ohm under an electro-motive-force of one volt.

An **Ohm** is a unit of resistance, or the resistance of a conductor through which a current of one ampere will pass under an electro-motive-force of one volt.

A **Watt** is a unit of power or energy, 746 watts = 1 h.p. A watt is the amount of electrical energy being used when one ampere of current is flowing under a pressure of one volt.

A **Kilowatt** is equal to 1000 watts.

A **Kilowatt-Hour** is equal to a kilowatt of electrical energy continued for one hour or its equivalent.

The resistance of conductors of the same material are directly in proportion to their length and inversely in proportion to their cross section. That is, doubling the length of a wire doubles its resistance, and doubling its diameter makes the resistance $\frac{1}{4}$ as great, since it makes the cross section 4 times as great.

1000 ft. of No. 10 B. & S. copper wire (0.102″ diameter) has an approximate resistance of 1 ohm.

Formulas:

$$C = \frac{E}{R}, \qquad E = C \times R, \qquad R = \frac{E}{C}, \qquad E \times C = \text{watts}.$$

E = electro-motive-force in volts.
R = resistance in ohms.
C = current in amperes.

Problem 1: How many amperes of current is flowing in a circuit with a resistance of 20 ohms when the pressure is 100 volts?

$$C = \frac{E}{R}, \qquad C = \frac{100}{20} = 5 \text{ amperes (Ans.)}.$$

Problem 2: What is the pressure when 5 amperes of current is flowing in a circuit in which the resistance is 20 ohms?

$$E = C \times R, \qquad E = 5 \times 20 = 100 \text{ volts (Ans.)}.$$

Problem 3: What is the resistance of a circuit when 5 amperes of current flow under a pressure of 100 volts?

$$R = \frac{E}{C}, \qquad R = \frac{100}{5} = 20 \text{ ohms (Ans.)}.$$

Problem 4: How many watts will be consumed, when a 10 ampere current is flowing under a pressure of 110 volts?

$$E \times C = \text{watts}, \qquad 110 \times 10 = 1100 \text{ watts (Ans.)}.$$

EXERCISES

1. (*a*) Find the resistance of 1 mile of No. 12 copper wire, if the resistance of 1 ft. of No. 12 wire is 0.002476 ohms.

(*b*) What length of wire will have a resistance of 125 ohms?

2. Find the resistance of 1000 ft. of No. 6 aluminum wire if 1 mile of this wire has a resistance of 3.3687 ohms.

3. What is the resistance of 100 ft. of German silver wire, if 11″ of this wire has a resistance of 0.022 ohms?

4. What is the resistance in ohms of a 110 volt line carrying a 10 ampere current?

5. Find the resistance of a 220 volt line carrying a 25 ampere current

6. What current in amperes is carried on a line that has a resistance of 0.8 ohms on a 110 volt circuit?

7. What ampereage is there in a 220 volt line which has a resistance of 50 ohms?

8. What kw. motor can be run with a 50 ampere current on a 220 volt line?

9. What e.m.f. is there on a line that has a resistance of 200 ohms, when a 10 ampere current is flowing?

10. What is the voltage of a battery that has a resistance of 0.25 ohms and a 6 ampere flow?

11. What e.m.f. exists between the ends of a wire whose resistance is 100 ohms, when the wire is carrying a current of 0.7 amperes?

12. How much current will flow between two points whose P.D. is 8 volts, if they are connected with a wire having a resistance of 350 ohms?

13. What will it cost per hour to operate a 5 ampere soldering iron on a 110 volt circuit. If 10 cents is being paid per kilowatt-hour.

14. What will it cost per year to burn five 40 watt lamps on an average of 3 hours per day. If 6 cents is paid per kilowatt-hour.

Horse Power Calculations, etc.

Work.—The unit of work is the foot pound or the amount of work in overcoming a pressure or weight equal to one pound through one foot space, and the time in doing this is not considered.

Formula: Weight in lbs. $\times$ distance in feet $=$ ft. lbs.

Example: If a man lifts a casting weighing 100 lbs. on a bench 3 ft. high, the work done is 300 ft. lbs. $100 \times 3 = 300$ (Ans.).

Power.—H.p. in mechanics is the power exerted or work done in lifting a weight of 33000 lbs. one foot per minute or 550 lbs. one foot per second. Time to do the work is always considered in h.p. The power of an average horse is about $\frac{3}{4}$ h.p. and that of an average man is about 1/7 h.p. Example: To lift a 3300 lb. casting 200 ft. high in 2 min., a 10 h.p. engine would be required.

Formula:

$$\text{h.p.} = \frac{\text{ft. lbs.}}{33000 \times \text{minutes}}.$$

$$\text{Solution} = \frac{3300 \times 200}{33000 \times 2} = 10 \text{ (Ans.).}$$

The Steam Engine

The power exerted by a piston driven by steam or other medium during the stroke in ft. lbs. is equal to the area of the piston times the pressure per square inch times the stroke in feet. In the case of steam engines, where the steam is cut off at $\frac{1}{4}$ or $\frac{1}{3}$ or $\frac{1}{2}$ of the stroke, the piston, being driven the rest of the way by the expansion of the steam, the average pressure for the entire stroke is the mean effective pressure (m.e.p.) as it is called, is the basis of calculation. As each revolution of the engine equals two strokes of the piston, the number of foot pounds per minute times length of stroke in feet, times r.p.m. times 2 = a product which divided by 33000 equals indicated h.p. (i.h.p.) or h.p. developed in cylinder. Example: An engine with a 10″ bore and a 12″ stroke running at a 100 r.p.m. with a m.e.p. of 80 lbs. will develop 38 plus h.p.

Formula 1:

$$\text{i.h.p.} = \frac{\text{area} \times \text{m.e.p.} \times \text{stroke} \times \text{r.p.m.} \times 2}{33000} = \frac{PLAN}{33000} \times 2.$$

Solution:

$$\frac{10 \times 10 \times 0.7854 \times 80 \times 1 \times 100 \times 2}{33000} = 38 + \text{h.p. (Ans.).}$$

Formula:

$$\text{Area of piston} = \frac{\text{i.h.p.} \times 33000}{\text{m.e.p} \times \text{stroke} \times \text{r.p.m.} \times 2}.$$

$$\text{Area} \times \text{stroke} = \frac{\text{i.h.p.} \times 33000}{\text{m.e.p.} \times \text{r.p.m.} \times 2}.$$

$$\text{m.e.p.} = \frac{\text{i.h.p.} \times 33000}{\text{area} \times \text{stroke} \times \text{r.p.m.} \times 2}.$$

Gas Engine H.P. Formulas

Brake h.p. = power actually delivered or i.h.p. minus friction of engine.

Formula 2: A.L.A.M. rating

$$\text{h.p.} = \frac{D^2 \times N}{2.5}.$$

D = Diameter of cylinder in inches.
N = No. of cylinders.

This formula is taken as a standard for 4 cycle single-acting engines at a piston speed of 1000 feet per minute.

Example: A 4 cylinder engine with $4\frac{1}{2}''$ bore will develop $32\frac{1}{2}$ h.p.

Formula 3:

$$\text{i.h.p.} = \frac{D^2 \times P \times L \times R \times N}{1{,}000{,}000}.$$

D = Diameter or bore of cylinder.
P = m.e.p. average about 80 lbs.
R = r.p.m.
N = No. of cylinders.
L = Length of stroke in inches.

Example: What is the i.h.p. of a 4 cyl. engine $4\frac{1}{2}''$ bore and $5\frac{3}{4}''$ stroke running at 1000 r.p.m. with a m.e.p. of 80 lbs.?

Solution:

$$\frac{20.25 \times 80 \times 5.75 \times 1000 \times 4}{1{,}000{,}000} = 37.26 \text{ i.h.p. (Ans.).}$$

EXERCISES

1. How much work in ft. lbs. will a man do per 10 hr. day, if he places 3800 bricks per hour on a wagon $2\frac{1}{2}$ ft. high, each brick weighing 86 oz.?

2. What force in lbs. does a man exert if he lifts a 1750 lb. casting 8 ft. high with a crane having a velocity ratio of 10 to 1?

3. What horse power does this man exert (above problem), providing he lifts the casting in 180 seconds?

4. A locomotive having two cylinders with a 26″ bore, 36″ stroke, running at 60 r.p.m. with a m.e.p. of 90 lbs. will develop what hp.?

5. How much work must be done to raise 120 long tons of coal from a mine 216 ft. deep? What must be the hp. of an engine to do it in four hrs. if the friction of the machinery increases the work 10 percent?

6. A cylindrical well 4 ft. in diameter and 72 ft. deep has 16 ft. of water in it. What must be the h.p. of the engine to empty this well in 40 minutes?

7. What must be the bore of an engine to develop 25 h.p. running 275 r.p.m. with a 12″ stroke and a m.e.p. of 60 lbs.?

8. What is the A.L.A.M. rating of a 2 cylinder gas engine having a 4″ bore?

9. What is the i.h.p. of a 4 cylinder gas engine running at 1200 r.p.m. having a $3\frac{1}{8}$″ bore and $3\frac{1}{2}$″ stroke, with a m.e.p. of 90 lbs. (using formula No. 3).

10. What h.p. will a 4 cylinder motor cycle develop running at 1400 r.p.m., with a 2″ bore and 3″ stroke, m.e.p. 80 lbs.? (Using formula No. 2 and No. 3.)

Strength and Proportion of Gear Teeth

The strength of gear teeth and the h.p. that may be transmitted by them depends upon so many variable and uncertain factors that sometimes it involves rather complicated formulas.

The various elements which enter in the constitution of a formula to represent the working strength are as follows:

1. The strength of material, which is often a variable quantity.
2. The shape of tooth, which is sometimes under-cut.
3. The point of application of the load (pitch line or extreme end). Some authorities differ on this subject.

4. Whether we consider the load applied to one or more teeth.
5. Pitch line velocity.
6. Factor of safety.

Formula 1—for cast iron gears:

$$\text{h.p.} = \frac{0.910 \times V \times P \times F}{\sqrt{1 + 0.65V}}.$$

V = velocity of pitch line in ft. per sec.
P = circular pitch in inches.
F = face of gear in inches.

The width of face is generally 2 to 3 times the circular pitch.

Formula 2:

$$\text{h.p.} = \frac{\text{pitch diameter} \times \text{r.p.m.} \times \text{C.P.} \times \text{face} \times 200}{126050}.$$

$$\text{C.P.} \times \text{face} = \frac{126050 \times \text{h.p.}}{\text{P.D.} \times \text{r.p.m.} \times 200}.$$

EXERCISES

1. What h.p. will the following gear safely transmit: 24″ P.D., 3.1416 C.P. 4″ face, running at 100 r-p.m.?

2. What h.p. can be transmitted with a cast iron gear $2\frac{1}{2}$″ P.D., 10 P, 1″ face, running at 500 r.p.m.?

3. What pitch gear must be used to transmit 1 H.P. with a cast iron gear 3.151″ P.D., running at 200 r.p.m., with a 1″ face?

4. What size face will be required on a gear 8″ P.D., 3 P, running at 350 r.p.m., to transmit 20 h.p.?

5. What pitch gear will be required on a hand crane, to be able to lift a 66000 lb. casting 10 ft. high in 10 minutes, the pinion having a 2″ face, 8″ P.D., being turned by a crank at the rate of 50 r.p.m.?

6. What must be the width of the face of a pinion 10″ P.D., 1 P, on a steam engine, running at 200 r.p.m., driving a derrick that is capable of lifting a 10 ton I beam 33 ft. in 1 min.?

7. What h.p. will a gear of 10 P, 2″ P.D., $1\frac{1}{2}$″ face, transmit running at 600 r.p.m.?

8. What h.p. can be safely transmitted with a 6 P gear, 6″ P.D., with a 1″ face, running at 250 r.p.m.?

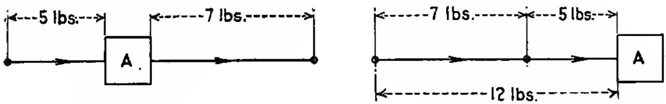
9. What width face will be required on a gear that is 20″ P.D., 2 P, running at 200 r.p.m., to transmit the maximum strain produced by a steam engine running at 275 r.p.m. with a 12″ stroke, 8″ bore and m.e.p. 120 lbs.?

10. A 10 h.p. engine driving a stone crusher through spur gears running at 150 r.p.m., having 10″ P.D., gears and 2″ face, will require approximately what pitch gears?

Resolution of Forces

Any single force which produces the same effect on a body as the combined action of two or more forces is called the **Resultant** of those forces. The resultant of two or more forces can be found either graphically or algebraically.

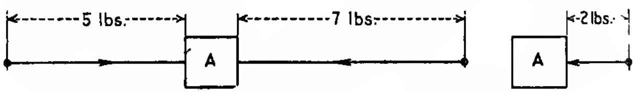
The resolution of two forces acting in the same straight line, in the same direction, is the sum of the given forces. Example: A force of 5 lbs. and 7 lbs. acting in an easterly direction on a body (*A*) will create a force of 12 lbs. in the easterly direction.



Graphical Representation. Resultant Force.

FIG. I

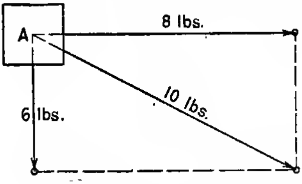
The resultant of two forces acting in the same straight line, but in opposite directions, is the difference of the given forces and act in the direction of the greater. Example: If there is a force of 5 lbs. on an object (*A*) in a westerly direction and a force of 7 lbs. acting in an easterly direction, the resultant force would be 2 lbs. in the easterly direction.



Graphical Representation. Resultant Force.

FIG. II

The resultant of any two forces acting at an angle to each other may be found by completing the parallelogram upon the forces as sides, and drawing the diagonal. Example: Suppose a force of 6 lbs. is acting on an object, (*A*) in an easterly direction, and a force of 8 lbs. in the southerly direction, then the resulting force will be equal to the diagonal, or 10 lbs., in the southeasterly direction.



Graphical Representation.
FIG. III

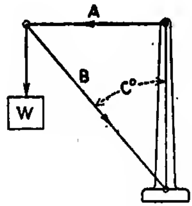


FIG. IV

Suppose a crane (Fig. IV) lifting a 1000 lb. weight: what would be the compression in lbs. on member *B*, and what would be the tension on tie rod *A*, if angle *C* was 60 deg. S.O. = 1000 sine of 30 deg. = 0.5000. Hyp. = S.O./sine or 2000 lbs. = compression on member *B*. S.O. × cotang. = S.A. or 1000 × 1.7321 = 1732.1 lbs. = tension on tie rod *A*.

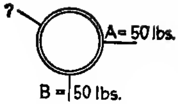
EXERCISES

1. If it requires two locomotives, one of 700 h.p. and the other 800 h.p. to pull a freight train, what is the total power in ft. lbs. per minute?

2. If there are 10 men, each pulling with a force of 60 lbs. on one end of a rope, and 12 men on the other end pulling with a 50 lb. force, what is the tension on the rope in ft. lbs., and what will be the direction of the resultant force?

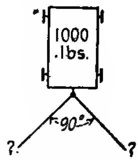
3. Two boys are rowing a boat in the middle of a river 1 mile wide, running north and south. "*A*" rows east at the rate of 3 miles per hour. "*B*" rows west at the rate of $4\frac{1}{4}$ miles per hour. Which boy and how long will it take him to reach shore?

4. If in Fig. IV, a crane is lifting an 8 ton weight, and the angle *C* is 50 deg., what will be the tension on *A* and compression on *B*.



5. Three ropes are fastened to a ring, and *A* is pulling on a rope with a force of 50 lbs. east. *B* is pulling on a rope with a force of 50 lbs. south. At what angle and with what force will a man have to pull on the third rope to keep the ring stationary.

6. A force of 20 lbs. is acting vertically upward and is resolved into two forces, one of which is horizontal and equal to 10 lbs. What is the direction and magnitude of the component?



7. If a casting on a truck requires a force of 1000 lbs. to move it, what two equal forces will be required to move it if the forces are acting at right angles to each other?

8. What will be the direction and magnitude of a force to equalize a force of 100 lbs. in an easterly direction and a force of 200 lbs. in a southerly direction?

9. What will be the direction and magnitude of a force to equalize a force of two tons in a southerly direction and a 3500 lb. force in an easterly direction?

10. What will be the pulling strain on a rope if a boy pulls with a force of 50 lbs. on one end, and two boys pulling in the opposite direction with a force of 25 lbs. each?

Falling Bodies

Gravity attracts all bodies toward the center of the earth with an acceleration which varies with the location of the body relative to the distance from the center of the earth. Theoretically acceleration is the same on all bodies, independent of their size, weight or shape.

A freely falling body, or a body moving under the influence of gravity (friction not considered), at the end of the first second, will have a velocity of 32.16 ft. per second. During the next second, it will acquire 32.16 ft. additional velocity, giving it a velocity of 64.32 ft. at the end of the second second. Each succeeding second will add 32.16 ft. to the velocity the body had at the end of the preceding second. This acceleration as it is called, due to gravity or 32.16 ft. in formulas, is designated by the letter "*g*."

V = velocity in feet per second.
g = acceleration due to gravity.
T = time in seconds.
S = space in feet.
H = height in feet.

To find the **Velocity** (V) of a falling body at the end of any number of seconds, multiply 32.16 by the number of seconds the body has fallen. Example: Find the velocity of a falling body at the end of 10 seconds.

Formula: $V = G \times T$. $V = 32.16 \times 10$, or 321.6 ft. (Ans.) or $V = 2S/T$.

The distance (S) through which a body falls in a given time equals the square of the number of seconds during which the body has fallen, multiplied by 16.08 or $\frac{1}{2}g$. Total distance fallen = S. Example: Find the total distance a body will fall in 10 sec.

Formula: $S = T^2 \times \frac{1}{2}g = 10^2 \times 16.08 = 1608$ ft. (Ans.) or $S = \frac{1}{2}VT$ or $S = V^2/2g$.

The time in seconds (T) required for a body to fall a given distance equals the square root of the distance expressed in feet, divided by 4.01. Example: Assume that a stone falls through a distance of 1608 ft. How long will it take to drop?

Formula:

$$T = \frac{\sqrt{S}}{4.01} = \frac{\sqrt{1608}}{4.01} = 10 \text{ sec. (Ans.) or } T = \frac{V}{g} \text{ or } T = \frac{2S}{V}.$$

The velocity (V) of a falling body after it has fallen through a given distance equals the square root of the distance through which it has fallen, times 8.02.

Example: What is the velocity of a falling body after it has fallen through a distance of 1608 ft.?

Formula: $V = 8.02 \times \sqrt{S}$. $8.02 \times \sqrt{1608} = 321.6$ ft. per sec. (Ans.) or $V = \sqrt{2gS}$.

The height (H) through which a body must fall to acquire a given velocity equals the square of the velocity divided by 64.32 or ($2 \times G$). Example: From what height must a body fall to acquire a velocity of 321.6 ft. per sec.?

Formula: $H = \frac{V^2}{2g} = \frac{321.6^2}{64.32} = 1608$ ft. (Ans.).

If a body is thrown vertically upward with a given velocity, its velocity will be retarded during each second in the same ratio as it is accelerated when falling. A body thrown vertically upward in the air will return to the ground with exactly the same velocity as that which it had when thrown up. At any point the velocity going up will be equal to the velocity going down except that the direction is reversed.

EXERCISES

1. What velocity has a body obtained after having fallen 5 seconds?

2. How far will a body fall in 15 seconds?—in 30 seconds?

3. How many seconds will it take for a body to fall 1000 ft.?

4. A body has fallen 100 ft. Find its velocity in ft. per sec. at the last sec.

5. How far must a body fall to acquire a velocity of 50 ft. per sec.?—100 ft. per sec.?

6. A person falls from a 22 story building which is 200 ft. high. What will be the velocity of the body? How long will it take the body to drop?

7. An object is thrown vertically upward with a velocity of 100 ft. per sec. When does it reach the highest point and start to return? How high will it go?

8. A body falls from a balloon to the earth in 10 seconds. What is the height of the balloon? What is the velocity of the falling body?

9. A falling body passes a given point (A) at a velocity of 15 ft. per second. How far below the point (A) is the body after 5 seconds? How far does the body fall during the fifth second after passing the given point (A)?

10. With what velocity must a ball be shot upward to rise to the height of the Washington Monument, which is 555 ft. high?

Centrifugal Force.

When a body revolves in a curved path, it exerts an outward force called **Centrifugal force** upon the arm or cords which restrains it from moving in a straight line. Example of this action, mud flying from a carriage wheel, bursting of emery wheels and fly wheels. This force depends upon the mass, velocity and radius of the revolving mass.

F = centrifugal force. R = radius in feet.
W = weight or mass. N = r.p.m.
V = velocity in feet per sec. G = gravity or 32.16.

Formulas:

$$F = \frac{WV^2}{GR}, \qquad R = \frac{WV^2}{FG}, \qquad N = \sqrt{\frac{2933F}{WR}},$$

$$W = \frac{FRG}{V^2}, \qquad V = \sqrt{\frac{FRG}{W}}.$$

Example: Find the centrifugal force or tension on the spokes of a fly wheel which is 6 ft. in diameter, running at 60 r.p.m., providing the rim weighs 300 lbs.

Formula:

$$F = \frac{WV^2}{GR} = \frac{300 \times (1 \times 6 \times \pi)^2}{32.16 \times 3} = 1104.8 \text{ lbs. (Ans.).}$$

For **Thin Disks** such as an emery wheel, rotating about its center, the sum of all radial or centrifugal forces that tend to rupture the disk equals $0.00000835WRN^2$.

EXERCISES

1. What is the centrifugal force on a fly wheel 10 ft. in diameter running 60 r.p.m., providing the rim weighs 500 lbs.?

2. A body weighing 200 lbs. is moving with a velocity of 40 ft. per sec. in a circle 10 ft. in diameter. What pressure is required to keep it in its path?

3. What will be the unbalanced strain on an automobile wheel, if there is a 1 lb. air valve on the rim 18″ from center, and the wheel running at 500 r.p.m. (about 55 miles per hour)?

4. What will be the strain on an emery wheel 24″ in diameter, weighing 50 lbs. running at 4000 surface ft. per minute?

5. What will be the maximum weight of a rim of a fly wheel, it being 4 ft. in diameter, running at 60 r.p.m., and the spokes are of such size that they can resist a strain of 600 lbs.?

6. What will be the maximum speed in ft. per min. that a fly wheel 6 ft. in diameter can be run, the rim weighing 400 lbs., with spokes of the same strength as in the above problem?

7. What diameter can we make a fly wheel within a safe limit, running at a surface speed of 3600 ft. per min., the rim to weigh 600 lbs. and the spokes to resist a strain of 2000 lbs.?

8. A locomotive weighing 100 tons, is running at a speed of 15 miles per hour. What side pressure will the rail receive when passing around a curve 120 ft. radius?

9. What will be the strain on an emery wheel 6″ in diameter, running at a surface speed of 5000 ft. per min., the wheel weighing 36 ounces?

10. A 10 lb. weight fastened on the end of a cord 5 ft. long is revolved at the rate of 100 r.p.m. What is the tension on the cord?

Horse Power of Belting

The h.p. a belt can safely transmit depends principally upon the velocity of the belt, the working stress or pull in pounds per inch of width the coefficient of friction and the arc of contact. The most economical speed for belting is between 4000 and 4500 ft. per min. In higher speeds than this, the action of centrifugal force is a hindrance. The most satisfactory working stress is about 575 to 850 lbs. per sq. in. of belt section. A commonly used value for the effective pull of a single belt is 35 lbs. per in. width, and 60 lbs. for double belt. A single belt is about 3/16″ thick and weighs approximately 16 oz. per sq. ft. A double belt is twice the thickness and weight. The ultimate strength of oak tanned leather runs from 3000 to 7000 lbs. per sq. in.

Rule 1.—A single belt 1″ wide running at 1000 ft. per min. will transmit 1 h.p.

Formula 1:

$$\text{h.p.} = \frac{P \times D \times N \times W}{4 \times 33000}, \qquad W = \frac{\text{h.p.} \times 33000 \times 4}{P \times D \times N},$$

Formula 2:

$$\text{h.p.} = \frac{PVW}{33000}, \qquad W = \frac{\text{h.p.} \times 33000}{PV}.$$

D = Diameter of driving pulley in inches.
V = Velocity of belt in ft. per min.
N = r.p.m. of driver.
P = Effective pull of belt per inch of width in lbs.
W = Width of belt in inches.

EXERCISES

1. What h.p. will a 6″ single belt transmit running at 4000 ft. per minute, using rule 1?

2. What width belt must be used to transmit 25 h.p. if the belt is running at 4500 ft. per minute, using rule 1?

3. What h.p. will a single belt 10″ wide transmit if it is running over two 10 ft. pulleys, running 110 r.p.m. What will a double belt transmit?

4. What width double belt must be used to safely transmit 50 h.p., if it is running at 4500 surface ft. per min., using formula 2?

5. What width single belt must be used to transmit 30 h.p. if the driver is 4 ft. in diameter, running at 400 r.p.m.?

6. A 10″ double belt driven by a 3 ft. driver, running at 300 r.p.m. will transmit what h.p.?

7. Using rule 1, what h.p. will a 5″ single belt transmit, running at 4500 surface ft. per minute?

8. According to rule 1, what width belt must be used to transmit 8 h.p. if the belt is running at 5000 surface ft. per minute?

9. A 10 h.p. stationary gas engine, running at 400 r.p.m., has a driving pulley 10″ in diameter. What must be the width of a double belt to safely transmit its full power?

10. What must be the width of a single belt to safely transmit the power of a $7\frac{1}{2}$″ h.p. motor, having a driving pulley 8″ in diameter running at 1500 r.p.m.?

Length of Belting

When Both Pulleys are of the Same Size.—Multiply the diameter of either one of the pulleys by π and add to this twice the distance between the shafts.

When One Pulley is Considerably Larger than the Other.—Square the distance between centers of the shafts, and add to this the square of the difference between the radii of the two pulleys. From this total, extract the square root, and multiply by 2 (calling this *A*). Add the diameter of the two pulleys and multiply by 1.57. Add this result to **A**, which will be the total length of belt.

Formula:

$$\text{Length} = [\{\sqrt{\text{C.D.}^2 + (R - r)^2}\} \times 2] + [(D + d) \times 1.57].$$

For Cross Belts.—

(*A*) Square the diameter of the large pulley and the distance between centers, add together and extract the square root.

(*B*) Square the diameter of the small pulley and the distance between centers, add together and extract the square root.

(*C*) To the sum of the two square roots, add the product of the sum of 2 pulley diameters × 1.57. The total will be the required length.

Formula:

$$\text{Length} = [\sqrt{(D^2 + \text{C.D.}^2)} + \sqrt{(d^2 + \text{C.D.}^2)}] + [(D + d) \times 1.57].$$

EXERCISES

1. What will be the length of the cross belt required to go over two pulleys 2 ft. and 4 ft. in diameter, shafts being 25 ft. apart?

2. What will be the length of belt to use over pulleys, both being 3 ft. in diameter, shafts 12 ft. apart? (For cross belt.)

3. Two pulleys 52″ and 24″ in diam., are on separate shafts 15 ft. between centers. What will be the length of belt required?

4. What is the length of cross belt running over two pulleys 16″ and 24″ in diameter, shafts being 20 ft. apart?

5. What is the length of cross belt 18 ft. between center of pulleys that are 8 ft. and 3 ft. in diameter?

6. What will be the length of belt required, if the center distance between shafts is 15 ft., the driver being 8″ in diameter, running at 1500 r.p.m. and the driven at 275 r.p.m.?

7. A gas engine has a drive pulley 10″ in diameter, running at 900 r.p.m., driving a line shaft 15 ft. away at 500 r.p.m. What will be the length of cross belt required?

8. Two pulleys 2 ft. and 5 ft. in diameter, on separate shafts 12 ft. apart will require what length of belt?

9. What will be the length of cross belt required for the above problem?

10. What is the length of cross belt, shafts being 30 ft. between centers, pulleys being 60″ and 48″ in diameter?

Rope Drives

Rope Drives are used for long distant transmission, and the effective pull of the rope depends upon the friction in the V shaped grooves on the pulley. These grooves are between 45 and 60 degrees.

Rope Pulley Diameters.—The diameter of the pulleys should not be smaller than 30 times the diameter of the rope to prevent internal wear, due to the fibers sliding upon each other.

Speed for Rope Transmission.—The rope should be run at a speed between 4000 and 5500 surface feet per minute. If they are run faster, the action of the centrifugal force is a hindrance. The tensile strength of manila rope in some cases is as high as 50,000 lbs. per sq. in. The strength of cotton rope is about four-sevenths of that of manila rope.

Center Distance depends upon the size of the pulleys and the size of the rope. This should not be less than 25 feet and not over 300 feet. If a longer drive is necessary, a series of drives may be arranged, or else the rope must be supported by loose pulleys to prevent excessive sag.

Ropes from 1″ to $1\frac{3}{4}$″ in diameter are generally used. The safe working stress for rope is about 200 lbs. per square inch of sectional area.

Formula: $D^2 \times 0.3$ = approx. weight in lbs. per linear ft. of rope.

Formula: h.p. $= D^2 \times V \times 0.003 \times N$.

D = diameter of rope in inches.

V = velocity in ft. per minute.

N = no. of ropes.

$$D = \sqrt{\frac{\text{h.p.}}{V \times 0.003 \times N}}. \qquad N = \frac{\text{h.p.}}{D^2 \times V \times 0.003}.$$

EXERCISES

1. What h.p. will a 1″ rope transmit, running at 5000 ft. per minute?
2. What h.p. will a 2″ rope transmit, running at 5000 ft. per minute?
3. Find the weight of 500 ft. of $1\frac{1}{4}$″ rope.
4. What is the weight per ft. of a $1\frac{1}{2}$″ rope?—of a 3″ rope?
5. An engine having a 20 ft. drive wheel, running at 80 r.p.m., driving a line shaft with 10—$1\frac{1}{2}$″ ropes will transmit what h.p.?
6. How many $1\frac{1}{4}$″ ropes must be used to transmit 100 h.p., if the surface speed is 4500 ft. per minute.
7. How many 1″ ropes are required to transmit 175 h.p. if the drive pulley is 10 ft. in diameter, running at 150 r.p.m.?
8. What will be the total weight of rope in the above problem if each rope is 125 ft. long?
9. I have a 50 kw.. motor running at 500 r.p.m. with a 3 ft. drive pulley. How many 1″ ropes should I use?
10. What size rope must be used to safely transmit the full power produced by a Corliss engine, having a 36″ stroke, 24″ bore, running at 150 r.p.m. with a m.e.p. of 125 lbs., providing 10 ropes are to be used on a drive wheel 10 ft. in diameter?

Cable or Wire Rope Drives

Wire Rope Drives are used for long distance transmission or where atmospheric conditions will not permit other methods. The effective pull of the wire rope depends upon the friction at the bottom of the grooves, the radius of which should be slightly larger than that of the rope. These grooves, are generally lined with leather, wood, rubber, or some other similar material to increase the friction between the rope and sheave and at the same time to reduce the wear of the rope to a minimum.

Cable Pulley Diameters should not be less than seventy five times the diameter of the cable.

Speeds for Cable Transmission should be between 4000 and 5000 surface feet per minute and the lower rope should be the one under tension to increase the arc contact.

Center Distance depends upon the size of pulleys and the size of cable. It should be between 60 and 400 feet. If a longer drive is necessary, the cable must be supported by loose pulleys to prevent excessive sag or a series of drives may be arranged.

Formula: $D^2 \times 1.58$ = approx. weight in lbs. per linear ft. of cable.

Formula: h.p. $= D^2 \times V \times 0.060 \times N$ for iron wire. For steel wire, multiply by 2.

$$\text{No. of cables} = \frac{\text{h.p.}}{D^2 \times V \times 0.060}. \qquad D = \sqrt{\frac{\text{h.p.}}{V \times 0.060 \times N}}.$$

EXERCISES

1. What h.p. will a 1″ iron cable transmit, running at 5000 ft. per minute?

2. What h.p. will two $\frac{5}{8}$″ steel cables transmit, running at 4000 ft. per minute?

3. Find the weight per ft. of a 1″ cable. Of a $1\frac{1}{2}$″ cable.

4. What is the weight of 700 ft. of $\frac{3}{4}$″ cable?

5. How many 1″ iron cables must be used to transmit 125 h.p., if the surface speed is 5000 ft. per minute?

6. What h.p. can be transmitted by a $\frac{7}{8}$″ iron cable, at 100 r.p.m. on a 5 ft. diameter drive sheave?

7. I have a 200 kw. motor running at 200 r.p.m., with an 8 ft. drive wheel. How many 1″ cables should I use?

8. How many $\frac{5}{8}$″ cables are required to transmit 200 h.p. if the surface speed is 4000 ft. per minute?

9. What size iron cables must be used to transmit the full power developed by a cross compound Corliss engine, having a 4 ft. stroke. The high pressure cylinder has a bore of 24″ and a m.e.p. of 80 lbs., and the low pressure cylinder has a 44″ bore and a m.e.p. of 25 lbs. Two ropes are used on a drive wheel 20 ft. in diameter, running at 65 r.p.m.

10. I have an 80 h.p. gas engine running at 300 r.p.m., with a 5 ft. drive pulley, driving a line shaft 50 ft. away at 200 r.p.m. What size steel cable must be used and what will be the weight of the cable?

Chain Transmission

Chain Drives are used for positive transmission of very heavy powers at rather slow speeds. There are several different types of chains, the round link chain used considerably on agricultural machines, the flat link and block chain as used on bicycles, and the silent chain. Chains should be well lubricated, or if possible run in oil.

Size of Sprocket Wheel.—The number of teeth in the sprocket should be as large as is consistent with other conditions. The best conditions are obtained with 16 or more teeth, although as few as 10 teeth can be used for slow speeds. The bottom diameter should be as accurate as possible, for on this diameter the chain rests. To obtain best results sprockets should always be machine cut.

Speed of Chain Drives should not exceed 2000 ft. per minute. Best conditions are obtained with a speed of 1000 ft. or under.

Center Distance should not be less than $1\frac{1}{2}$ times the diameter of the large sprocket, nor more than 12 ft. If longer drives are necessary, a series of drives should be arranged; or else the chain must be supported by loose pulleys to prevent excessive sag, which causes the chain to whip. The center distance should be adjustable if possible, to regulate the amount of sag.

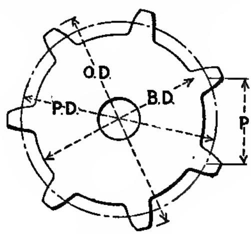


FIG. I

The **Safe Working Load** of a chain depends on the amount of rivet bearing surface, and varies from 1/10 to 1/40 of the tensile strength, according to the speed, size of sprocket, etc.

Chain Sprocket Diameters, etc.

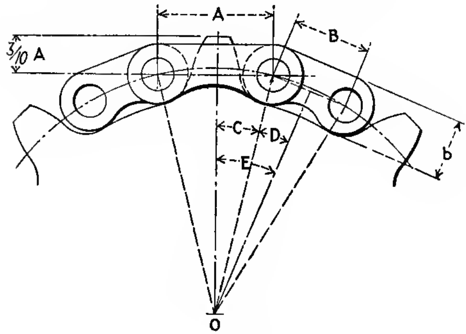


FIG. II

P = Pitch.
N = Number of teeth.
b = Diameter of round part of chain block (.325″ for 1″ P, and .532″ for $1\frac{1}{2}$″ P).
B = Center to center of holes in chain block (.400″ for 1″ P, and .564″ for $1\frac{1}{2}$″ P).
A = Center to center of holes in side bars (.600″ for 1″ P, and .936″ for $1\frac{1}{2}$″ P).

$$\text{Tan } C = \frac{\text{Sin } \frac{180°}{N}}{\frac{B}{A} + \text{Cos } \frac{180°}{N}}. \qquad \text{Pitch diameter} = \frac{A}{\text{Sin C}}.$$

$$\text{O.D.} = \text{P.D.} + b. \qquad \text{B.D.} = \text{P.D.} - b.$$

To Calculate Length of Chain

As a chain cannot contain a fractional part of a pitch, the next whole number above the calculated number of pitches is generally used. The chain length in inches is found by multiplying the number of pitches by the pitch in inches.

EXERCISES

1. What is the P.D. of a 10 tooth sprocket wheel with a 1″ pitch chain?

2. What is the O.D. and B.D. of an 18 tooth sprocket with a 1″ pitch chain?

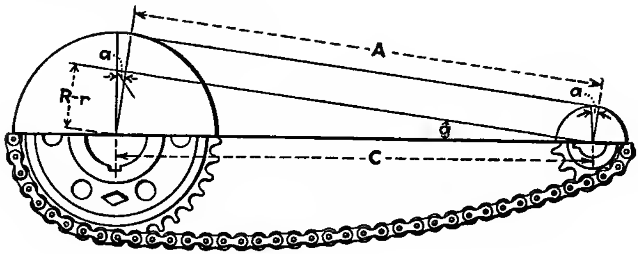


FIG. III

P = Pitch of chain.
C = Center distance in pitches.
N = Number of teeth on large sprocket.
n = Number of teeth on small sprocket.

$$L = \text{Chain length in pitches} = 2C + \frac{N}{2} + \frac{n}{2} + \frac{0.0257(N - n)^2}{C}.$$

3. What is the distance A–B and diameter b of a $1\frac{3}{4}$″ pitch chain? (using the same proportions as a 1″ pitch chain).

4. Find the P.D., O.D. and B.D. of a 30 tooth sprocket for a $1\frac{1}{2}$″ pitch chain.

5. Find the B.D. of a 16 tooth sprocket for a $1\frac{1}{2}$″ pitch chain.

Shaft Design

Shafting is generally made of cold rolled steel (C.R.S.), and is subject to two stresses: Shear stress due to the tension of the belt and weight of the shaft and pulleys: and twisting or torsional stress produced due to the resistance of the load.

Shafting is generally calculated for torsional stress only although for light shafting with heavy pulleys, etc., bending stresses must be taken into consideration.

Pulleys should be fastened as close as possible to the bearing to prevent excessive deflection.

The **Allowable Stress** for shafting varies from 5000 to 9000 lbs. per sq. in.

H.p. formulas:

For long main power transmission shafts

$$\text{h.p.} = \frac{D^3N}{80}. \quad \text{Diameter} = \sqrt[3]{\frac{80 \text{ h.p.}}{N}}.$$

For regular line shafts carrying drive pulleys

$$\text{h.p.} = \frac{D^3N}{54}. \quad \text{Diameter} = \sqrt[3]{\frac{54 \text{ h.p.}}{N}}.$$

For small short shafts well-supported

$$\text{h.p.} = \frac{D^3N}{40}. \quad \text{Diameter} = \sqrt[3]{\frac{40 \text{ h.p.}}{N}}.$$

Angle of torsional deflection for round shafts:

$$A = \frac{583.6TL}{D^4E}.$$

The torsional deflection of round shafts is 70% greater than that of a square one.

A = angle of deflection in degrees.

D = diameter of shaft in inches.

E = torsional modules of elasticity about 12,000,000 for steel shafting.

L = length of shaft in inches.

N = r.p.m.

T = twisting moment in inch pounds.

Torsional deflection of shafting should not exceed 5 min., or about 0.08 deg. per linear ft.

$$D = 4.6\sqrt[4]{\frac{\text{h.p.}}{N}}.$$

Linear deflection or sag should not exceed 0.012″ per linear ft. The maximum distance in feet between bearings in order to prevent excessive sag should be calculated according to the following formula:

$$L = \sqrt[3]{140D^2}.$$

EXERCISES

1. Find the torsional deflection of a 3″ shaft 6 ft. long, subject to a twisting moment of 25000 in. lbs.

2. What should be the diameter of a line shaft to transmit $7\frac{1}{2}$ h.p., shaft running at 275 r.p.m.?

3. What h.p. will a 2″ shaft transmit, shaft running at 350 r.p.m.? (using formula for short shafts).

4. Find the diameter of a line shaft to transmit 50 h.p. at 100 r.p.m., with a torsional deflection not to exceed 0.08 deg. per linear ft.

5. What should be the maximum length between bearings for a 1″ shaft?—for a 2″ shaft?

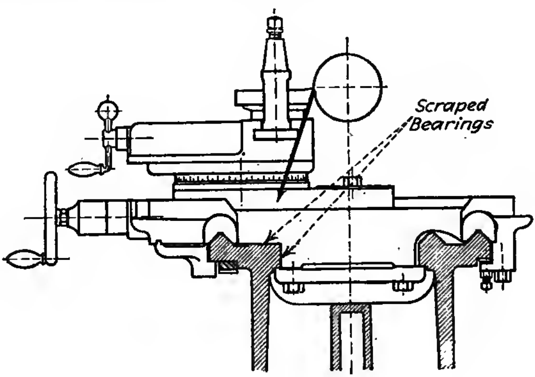
6. Find the h.p. that can be transmitted by a $1\frac{1}{2}$″ shaft, at 150 r.p.m.? (using formula for short shafts).

7. What diameter shaft is required to transmit 700 h.p., running at 125 r.p.m.?

8. Find the torsional deflection of an 8″ shaft 10 ft. long, subject to a twisting moment of 20,000 in. lbs.

9. What should be the maximum distance between bearings for a 3″ shaft?

10. Find the diameter of line shaft to transmit 500 h.p. at 200 r.p.m., with a maximum torsional deflection of 0.07 deg. per linear ft.



Bearing Design

The **Size of Bearing** required depends upon the bearing material, quality of oil used, speed, bearing pressure running conditions, etc.

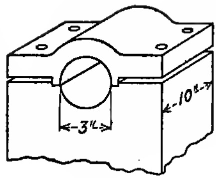


FIG. I

The **Projected Area of a Journal** (Fig. I) is the size used in calculation of the bearing surface; thus, the area of bearing surface for a 3″ × 10″ journal is 3″ × 10″ or 30 sq. in.

The **Amount of Pressure** (P) allowed per sq. in. of projected area of bearings varies from 400 to 2000 lbs.

The **Diameter** of shaft or pin must be designed strong and rigid enough to carry the required load. To do this the approximate length must be known.

The **Length of Bearing** must be designed to have the proper bearing surface so that the unit pressure shall not exceed the allowable load. In general practice the length of bearing is from one to three times the shaft diameter.

Formulas:

A = allowable pressure per sq. in. of projected area of bearing.

D = diameter of bearing in inches.

Y = quantity depending on method of oiling $\begin{cases} \text{drop feed 800,} \\ \text{force feed or ring oil 1400,} \\ \text{best conditions 2000.} \end{cases}$

L = length of bearing in inches.

N = r.p.m.

P = maximum safe unit pressure $\begin{cases} \text{400 to 600 for shaft bearings,} \\ \text{800 to 1000 for car journal,} \\ \text{1200 to 1400 for crank pins,} \\ \text{1600 to 2000 for wrist pins} \\ \quad \text{and sliding shoes.} \end{cases}$

W = total load upon bearings in lbs.

$$A = \frac{PY}{(DN) + Y}.$$

For bearings in the form of a sliding shoes or ways, the

quantity $250V$ is substituted for the quantity DN in the formula. V is the velocity of the rubbing surface in ft. per sec. To find the proper length of bearing use the following formula:

$$L = \frac{W}{PY}\left(N + \frac{Y}{D}\right).$$

If the above formula gives a journal too long for practical use, the proper proportion of diameter and length should be adjusted to meet the required conditions.

If the required projected area is known, the following formula may be used to get the proper proportion of the length to the diameter: $L : D : : (\frac{1}{8}\sqrt{N}) : 1$.

EXERCISES

1. What will be the allowable pressure per sq. in. on a car journal, using drop feed lubrication, if the journal is 4″ in diameter, running at 80 r.p.m.?

2. What length bearing will be required in the above problem if there are 8 such bearings under a box car which weighs 18 tons and has a carrying capacity of 80,000 lbs.?

3. Design a main bearing and an outboard bearing for an engine, providing a 16″ shaft is to be used. The weight of the flywheel, shaft, crank-pin and $\frac{1}{2}$ of connecting rod and other movable parts supported by bearings weigh 100,000 lbs. and 3/5 of this weight comes on the main bearing, and the remainder on the outboard bearing, the engine running at 100 r.p.m. (Ring oiling system to be used.)

4. What length of crank pin bearing must be used for an engine with a 10″ bore, running at 100 r.p.m., maximum steam pressure of 125 lbs. and force feed lubrication, if the crank pin is $2\frac{3}{4}$″ in diameter?

5. The $2\frac{1}{16}$″ diameter line shaft in Dept. T-20 is 62 ft. long, with bearings 10 ft. apart, total weight of 27 pulleys and belt strain being approximately 3360 lbs. What should be the length of the journals used, if the shaft turns 280 r.p.m.?

6. What should be the length of 2 journals on an electric motor running at 1190 r.p.m., the armature, commutator, pulley, belt strain and shaft weigh 1500 lbs., providing the shaft is 2″ in diameter, using ring oil lubrication?

7. What should be the bearing surface on the V's of a planer table if the table and work weighs 45000 lbs., and the maximum pressure produced by cutting is 700 lbs., with a maximum rate of table travel of 2 ft. per second, using ring oil lubrication and allowing a factor of safety of 4?

8. If 24 square inches is required for project area of a bearing, what will be its size according to formula $(L : D :: (\frac{1}{8}\sqrt{N}) : 1)$, if the number of revolutions per min. is 300.

9. What should be the length of two bearings on a lathe if an $1\frac{1}{2}''$ shaft is used, and the weight of cone pulley, shaft, and pressure of maximum cut equals 1000 lbs., providing 3/5 of the pressure is on the front bearing, and the maximum number of revolutions per min. will be 1200?

10. What must be the length of two bearings to support a 10 ton fly wheel on a $7''$ shaft running at 150 r.p.m., providing the wheel is 2 ft. from the center of one bearing and 3 ft. from the center of the other bearing, if force feed lubrication is to be used?

Ball Bearing Design

Ball Bearings can be divided into 3 groups; **Radial,** those that carry a radial load; **Thrust,** those that take a thrust or end load and the **Combination Radial and Thrust,** which take both a radial and thrust load.

Ball bearings are especially adapted for high speed and light running machines. They are used in preference to sliding bearings due to the following reasons: less loss of power on account of the smaller coefficient of friction, less danger of bearing heating, and also because shorter bearing can be used.

Rolling friction is always less than sliding friction.

Friction of balls is independent of viscosity of the lubricant used.

Ball bearings should always be lubricated, preferably with hard oil. Nothing but tool steel or alloy steel should be used for high grade work although case hardened machine steel balls are sometimes used for large balls and light running conditions.

The carrying capacity of a ball bearing is directly propor-

tional to the number of balls and to the square of the ball diameter.

The **Permissable Load** that annular ball bearing will carry can be determined approximately by the following formulas:

$$P = 0.65 \times Y \times D^2 \times N, \qquad N = \frac{P}{0.65 \times Y \times D^2},$$

$$D = \sqrt{\frac{P}{0.65 \times Y \times N}},$$

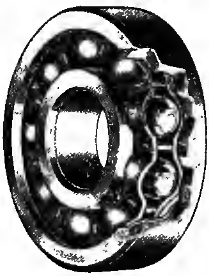


FIG. I. RADIAL TYPE

P = load capacity in pounds,
D = diameter of balls, taking $\frac{1}{8}''$ as a unit of diameter. Example ($\frac{3}{8}''$ $D = 3$),
N = number of balls,
Y = constant which varies with conditions and type of bearings, also material and speed.
Y = 10 for 500 r.p.m.
Y = 7.5 for 1000 r.p.m.
Y = 5 for 1500 r.p.m.

EXERCISES

1. What is the allowable load on a ball bearing having 10—$\frac{1}{4}''$ balls on a shaft running at 1500 r.p.m.?

2. How many $\frac{1}{2}''$ balls are necessary in a ball race to carry a load of 1,000 lbs.?—for 2,000 lbs.? (Shaft running at 1,000 r.p.m.)

3. What size balls must be used to carry a load of 300 lbs. providing 10 balls are to be used, shaft running at 1,000 r.p.m.?

4. What size balls must be used in bearings of an electric motor if total load on the two bearings is 700 lbs., 12 balls to be used in each bearing and motor is running at 1500 r.p.m.?

5. What size balls are required in each bearing if the load consists of a 900 lb. fly wheel, and a 267 lb. shaft running at 1,000 r.p.m. providing the fly wheel is 5 ft. from one bearing, and $2\frac{1}{2}$ ft. from the other, and 50 balls are to be used in each bearing?

6. How many $\frac{11}{16}''$ balls can be placed around a shaft which is 2″ in diameter?

7. What size balls must be used if I want to put 11 balls around a shaft 2.550″ in diameter?

8. How many $\frac{1}{4}$″ balls can be put in a bearing if the outer ball race is 1.456″ in diameter?

Centers of Gravity and Gyration, and Moment of Inertia, etc.

Center of Gravity of a body is the point about which, if suspended, all the parts will be in equilibrium, that is, there will be no tendency to rotate. The center of gravity of a figure, symmetrical about a center line, lies on that line. To find the center of gravity of any figure, draw a line $B-C$ (Fig. I), and perpendicular limited lines $B-E$ and $F-C$. Divide $B-C$ into any number of equal parts, and erect perpendiculars a, b, c, d, etc. at their middle points.

The area of each section is the product of the length of middle line multiplied by width of section.

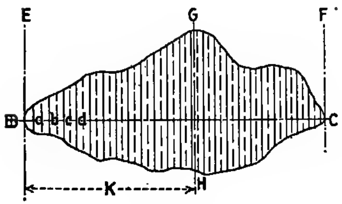


FIG. I

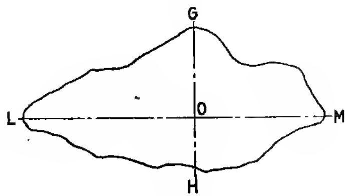


FIG. II

The **Moment of Area** about the axis $E-B$ is found by multiplying each area by the distance from the axis $E-B$ to the middle line.

The sum of the total moment of area divided by the total area is equal to K, or the distance from axis $E-B$, to the center of gravity of the area.

Sum of moments of area = total area times K, or

$$K = \frac{\text{sum of moments of area}}{\text{total area}}.$$

By turning the figure 90 deg. and repeating the process, L–M (Fig. II) is found. The intersection of these lines is the center of gravity of the whole figure. The greater the number of divisions, the more accurate will be the result obtained. The center of gravity of an irregular shaped body which lies in a plane, is found by the same method.

$$K = \frac{\text{Sum of all products of each weight of section multiplied by distance}}{\text{total weight}} .$$

The center of gravity of a body which does not lie in a plane may be found by taking the moment about 3 planes which are at right angles to each other.

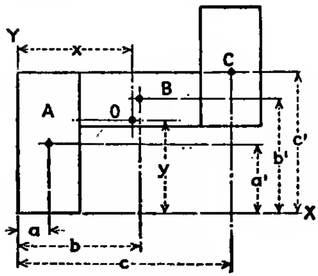


Fig. III

The **Center of Gravity of a Figure** if composed of several rectangles, triangles, etc. can easily be found in the following manner:

If A, B, C = area of rectangle; a, a', b, b', c, c' = distance the center of gravity of each rectangle is from axis X and Y, which was conveniently selected; x, y = distance the center of gravity of whole figure is from axis X and Y. Then

$$x = \frac{(A \times a) + (B \times b) + (C \times c)}{A + B + C},$$

$$y = \frac{(A \times a') + (B \times b') + (C \times c')}{A + B + C}.$$

Then the point of intersection of the lines at O = center of gravity.

The center of gravity of any triangle is located at the intersection of lines drawn through $\frac{1}{3}$ of the altitude, and parallel with the bases.

The Moment of Inertia of a body (or section) with respect to an axis, is the sum of the products obtained by multiplying the weight (or area) of each elementary particle by the square of its distance from the axis. It is generally represented by the symbol I. The value of it varies according to the position of axis. The moment of inertia is smallest when the axis passes through the center of gravity.

The moment of inertia of any section about any axis is equal to its moment of inertia about a parallel axis through the center of gravity, plus the product of the area and square of the distance between the axis.

If I' = Moment of inertia about an axis through center of gravity of section.
I = Moment of inertia about any parallel axis.
A = area of cross-section.
r = distance between axis of I' and I.
Then $I = I' + A \times r^2$.

The value of moment of inertia of various sections about an axis, passing through center of gravity of section, is given in table V, (page 114).

Example: Find the moment of inertia of the section of a beam shown in Fig. IV about the horizontal axis E–F through the center of gravity. Divide the figure into 3 rectangles A, B, C. The center of gravity of each rectangle will lie on a line running through the middle height. Therefore

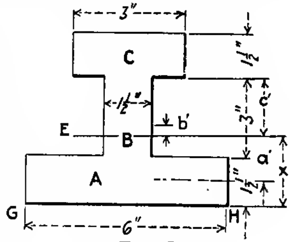


FIG. IV

distance of A from G–H = $1\frac{1}{2} \div 2 = \frac{3}{4}''$.
distance of B from G–H = $1\frac{1}{2} + 3/2 = 3''$.
distance of C from G–H = $1\frac{1}{2} + 3 + \frac{3}{4} = 5\frac{1}{4}''$.

$$\begin{aligned} \text{Area of } A &= 1\tfrac{1}{2} \times 6 = 9 \text{ sq. in.} \\ \text{Area of } B &= 3 \times 1\tfrac{1}{2} = 4\tfrac{1}{2} \text{ sq. in.} \\ \text{Area of } C &= 1\tfrac{1}{2} \times 3 = 4\tfrac{1}{2} \text{ sq. in.} \\ \text{Total area} &= 18 \text{ sq. in.} \end{aligned}$$

$$x = \frac{\text{sum of moments}}{\text{total area}}$$

$$= \frac{(9 \times \frac{3}{4}) + (4\frac{1}{2} \times 3) + (4\frac{1}{2} \times 5\frac{1}{4})}{18} = 2\ 9/16''.$$

Distance of center of gravity A from $E\text{–}F$ (a')

$$= 2\ 9/16'' - \tfrac{3}{4}'' = 1\ 13/16''.$$

Distance of center of gravity B from $E\text{–}F$ (b')

$$= 3'' - 2\ 9/16'' = 7/16''.$$

Distance of center of gravity C from $E\text{–}F$ (c')

$$= 5\tfrac{1}{4} - 2\ 9/16'' = 2\ 11/16''.$$

Moment of inertia of rectangle about its neutral axis (axis through center of gravity) is $bd^3/12$ (from table V, page 114).

$$\text{Therefore moment of inertia of } A = Ia = \frac{6 \times (1\frac{1}{2})^3}{12} = 1.69,$$

$$\text{moment of inertia of } B = Ib = \frac{1\frac{1}{2} \times (3)^3}{12} = 3.38,$$

$$\text{moment of inertia of } C = Ic = \frac{3 \times (1\frac{1}{2})^3}{12} = 0.84.$$

Moment of inertia about the new axis $E\text{–}F$ is:

$$\begin{aligned} I'a &= Ia + 9 \times (1\ 13/16)^2 = 1.69 + 29.51 = 31.20. \\ I'b &= Ib + 4\tfrac{1}{2} \times (7/16)^2 = 3.38 + 0.86 = 4.24. \\ I'c &= Ic + 4\tfrac{1}{2} \times (2\ 11/16)^2 = 0.84 + 30.55 = 31.39. \end{aligned}$$

Required total moment of inertia

$$= I'a + I'b + I'c = 66.83 \text{ (Ans.)}.$$

The **Center of Gyration** with respect to an axis, is a point at which, if the entire weight of a body is concentrated, its moment of inertia will remain unchanged; or in a revolving

body the point in which the whole weight of the body may be considered to be concentrated.

Radius of Gyration is the distance from the axis to the center of gyration.

I = moment of inertia of a body.
I' = moment of inertia of a section.
W = weight of the body.
G = radius of gyration.
A = area of section.

$$I = W \times G^2.$$

$$I' = A \times G^2.$$

The value of radius of gyration for several sections can be found from table V (page 114).

EXERCISES

1. Find the distance of center of gravity of an angle section 6″ × 4″ × 1″, from the back of each leg (Fig. I).

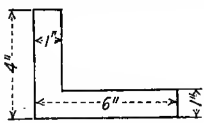


FIG. I

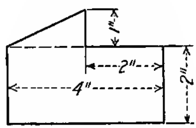


FIG. II

2. Locate the center of gravity of the figure shown in Fig. II, taking the origin at the left hand corner of the bottom.

3. A rectangular board 10″ wide and 12″ high has a circular hole cut out. Center of hole is 4″ from the right edge and 4″ from the bottom. Its diameter is 3″. Locate the center of gravity of the remainder, taking origin at the lower left hand corner.

4. Find the moment of inertia of the angle of problem No. 1, with respect to an axis parallel to its base through its center of gravity.

5. Also of Fig. II.

PART IV

Graphical Charts

In many instances records, statistics, data, etc., of industrial organizations are kept by means of graphical charts, which provide a quick and accurate method of tabulating; same also being valuable to the busy executive, who can find at a glance, the information desired.

Charts of this nature are also used frequently by statisticians, engineers, production managers, and experimental laboratories for keeping records, plotting values, etc.

Graphical charts may be constructed in many forms, depending upon the nature of the work to be tabulated.

The following are a few of the methods generally used and should prove of value to those who desire to become familiar with keeping records graphically.

Fig. I shows a chart which is frequently used in keeping track of production. The chart is outlined in advance according to the production schedule desired; for instance, if the management decides that they desire to manufacture 3000 machines between the months of January and October inclusive. The chart is divided into months as shown horizontally together with the number of parts desired per month. In this chart each month is divided into 3 divisions representing 100 parts each, although any other convenient scale may be used. On the left hand side and vertically are placed the name and number of the parts. As the parts are machined in the shop the production clerk blocks in the space opposite each part, thus the length of the blocked line after each part shows at a glance the total production up to date.

For instance in the month of April 1200 gears No. 3876

have been made which is just even with the advanced schedule, while only 1100 levers No. 4401 have been made showing a shortage of 100 thus it will be necessary to speed up production on this part. The chart shows that 2400 pins No. 5628 have been made and as this is sufficient to last until August thus the machine or machines making these pins can be set to work on some other parts for a few months.

Charts of this form are usually fastened to a board on the wall and are filled in by means of a color crayon or in some cases tacks are moved along a line opposite the parts as the production advances. The location of the tacks on the chart shows the maximum production up to date.

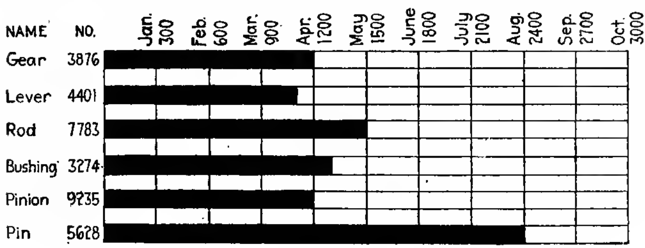


FIG. I

Fig. II is also an advance production chart although it can be used for many other purposes.

Thus if a concern decides to make 1200 adding machines in one year a chart may be plotted as shown; lines representing months are drawn vertically and lines representing quantities of 100 per division, more or less as desired, are drawn horizontally. A line connecting the lower left hand point with the upper right hand point will be the theoretical production line. As shown January production was 30 machines, February 70 machines, thus the sum of the months production is always added to the total sum of the production up to the preceding month to obtain the location

of the production point for the month in question. 70 machines being made in February the production point will be 30 plus 70 equals 100 machines or total production up to the end of February. In March 75 machines were made, thus 100 plus 75 equals 175 or the location of the production point for March. By placing the production points in for the various months and connecting same a production curve will be generated, which will show the number of machines made each month, the total number of machines made up to any month and it will also show how close the actual production curve follows the theoretical one.

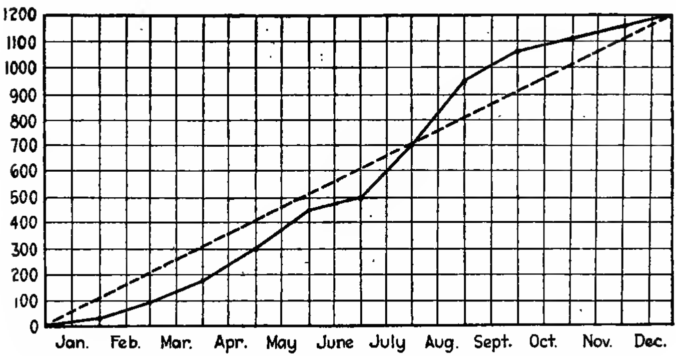


FIG. II

Fig. III shows a monthly chart which is laid out for a daily record of the number of cars assembled and shipped in a large automobile plant. The body of the chart is laid out the same as Fig. II after which the daily reports can be located on the chart and lines connecting these points will be a broken curve showing the daily fluctuation.

To distinguish one curve from another a combination of broken lines, as shown, can be used or if desired inks of various colors can be used which will give a pleasing contrast.

Fig. IV is a chart somewhat different from the preceding ones and is very simple and easy to understand. It can be laid out vertically or horizontally as desired with the overall length representing the whole or 100%. Then, by dividing the overall length into lengths of such proportion as are desired and shading or coloring each proportion differently a very effective chart is produced.

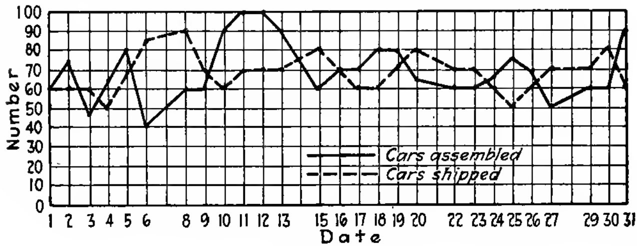


FIG. III

This chart shows the proportion of the distribution of heat in a heating plant.

Thus 16% of the heat produced by burning coal escapes through the chimney, 2% by radiation, 3% carbon in the ashes, 2% by moisture in the coal, $2\frac{1}{2}$% unaccountable losses and the remainder or $74\frac{1}{2}$% of the heat goes to heat the water in the boilers or called efficiency.

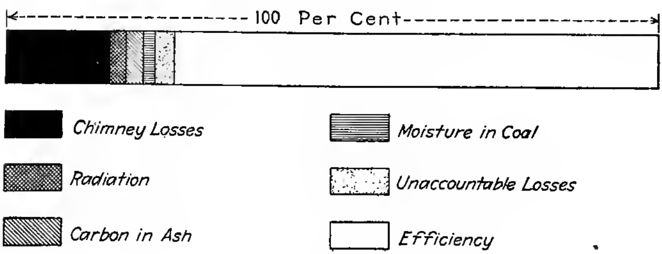


FIG. IV

Fig. V is a rather peculiar form of chart which is easily constructed and shows the distribution of the units very effectively. The proportion of the chart that is of most importance is usually turned to face in one direction as shown (25% work) while the remainder of the proportions face in the opposite direction.

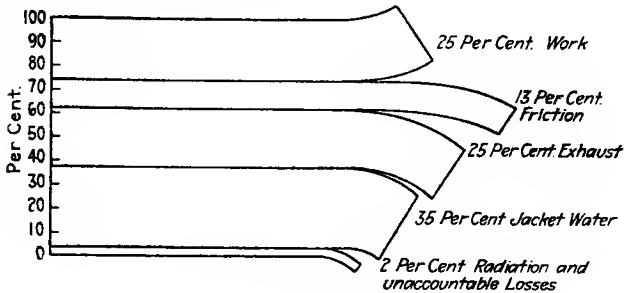


FIG. V

This chart shows the distribution of heat in the average gasoline engine. Thus the total heat as developed by burning fuel is distributed as follows: 25% work, 13% friction, 25% exhaust, 35% through the water which circulates around the cylinders, and 2% due to radiation and unaccountable losses.

Fig. VI is a chart which was used in a press room although applicable in a variety of ways, so that the superintendent could tell the proportion of time the various presses were in

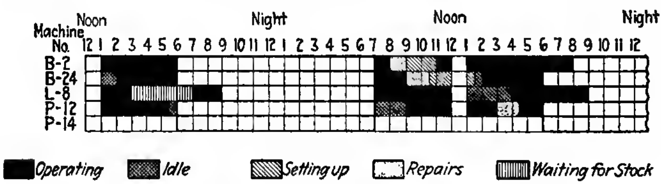


FIG. VI

actual operation or whether they were idle, or inoperative due to setting up, repairs or waiting for stock.

The so-called overtime or night portion of the chart was underlined heavily, as shown, so that any overtime would show up more distinctly as it was desired to keep the overtime work down to a minimum.

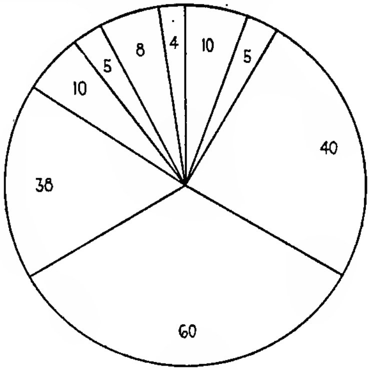


FIG. VII

No.	Operation	Time
1.	Place in chuck	10
2.	Start machine	5
3.	Face off ends	40
4.	Turn diameter	60
5.	Cut groove	38
6.	File off corners	10
7.	Stop machine	5
8.	Gauge piece	8
9.	Remove from chuck	4
	Total =	1.80

Fig. VII is a circular form of chart which is used quite extensively for graphical representation of proportions. The circumference is usually subdivided into the proportions desired, after which lines are drawn from these points toward the center. These wedge shaped divisions afterward being

numbered showing the unit of time or percent which they represent. Frequently the wedge shaped divisions are colored in various shades to bring out distinction.

Fig. VIII is another form of circular chart although in reality it is form of chart similar to Fig. III developed around a common center. This chart was used by an employment manager and showed the total number of men hired per month. As shown in the chart, 50 men were hired during the month of January, 65 in February, 60 in March, etc.

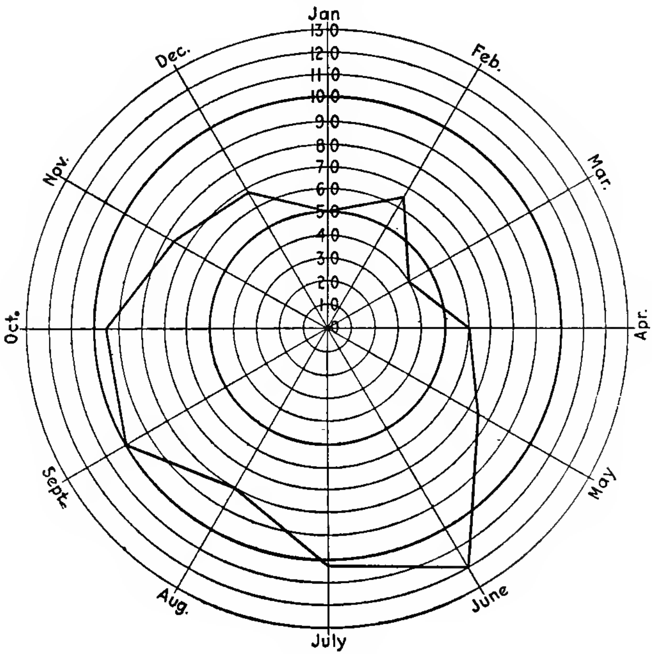


FIG. VIII

Fig. IX is a chart plotted from a well-known 6 cylinder

automobile gas engine having a $3\frac{1}{4}''$ bore and a $5''$ stroke to show the horse power and torque in foot pounds at different speeds ranging from 200 to 2400 r.p.m., equivalent to a car velocity of 4.3 to 51.7 miles per hour respectively.

As shown, the h.p. and torque were plotted at steps of an increase of 200 revolutions each and a curve drawn through these points gave the h.p. and torque curve from which the h.p. or torque can be obtained for all intermediate speeds.

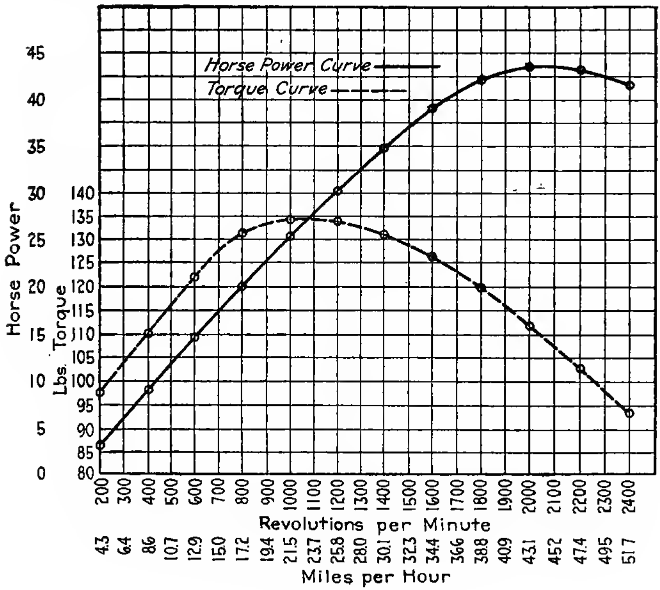


FIG. IX

EXERCISES

1. Draw a production chart similar to Fig. I for the months of January to August inclusive for 4 parts, a crank, gear, axle and connecting rod, if the desired output is 500 parts of each per month. In the chart show a production of a full number of cranks for the month of May, 300 gears and 900 axles in excess for May and 250 behind schedule on connecting rods.

2. Lay out a chart similar to Fig. II showing a theoretical production curve for a period of 6 months, if it is desired to make 12,000 three inch shells. Also show the actual production curve from the following data: January 500, February 1600, March 1800, April 2500, May 2000 and June 3600.

3. Make a monthly chart for an employment office similar to Fig. III for a period of one year, showing two curves, one for skilled and the other for unskilled labor, if the following number of each were hired per month.

Month	Skilled	Unskilled	Month	Skilled	Unskilled
Jan.	28	22	July	100	120
Feb.	30	40	Aug.	82	98
Mar.	16	28	Sept.	68	110
Apr.	40	50	Oct.	54	80
May	70	38	Nov.	50	63
June	60	78	Dec.	38	71

4. Lay out a diagram similar to Fig. IV showing the proportion of the number of various automobiles as stated below in the state of Michigan.

Fords—40 percent
Buick—7 percent
Overlands—6 percent
Studebaker—5 percent
Reo—4 percent
Maxwell—3 percent
Cadillac—3 percent
All others—32 percent

5. Draw a diagram similar to Fig. V showing the proportion of the various substances as shown by the following analysis of the average run of Pennsylvania anthracite. Free carbon 82.5 percent, ash 9.0 percent, volatile matter 5.0 percent and moisture 3.5 percent.

6. Lay out a chart similar to Fig. VI showing the average time you spend per 24 hours at the following: sleep, meals, recreation work and idleness.

7. Make a circular chart similar to Fig. VII showing the ratio of the proportions of the following analysis of a bearing alloy: copper 74 percent, zinc 11 percent, lead 10 percent and tin 5 percent.

8. Make a chart similar to Fig. VIII showing the average monthly temperature for a period of one year from the following data:

Jan. −15°	Feb. −8°	March −2°	April 12°	May 40°	June 72°
July 86°	Aug. 92°	Sept. 80°	Oct. 76°	Nov. 70°	Dec. 35°

9. Plot a horse power and torque curve similar to Fig. IX from the following data obtained from a 6 cylinder gas engine in a dynamometer test.

R.P.M.	H.P.	Torque in Ft. Lbs.	R.P.M.	H.P.	Torque in Ft. Lbs.
200	2.5	95	1400	36	131
400	7	115	1600	38	126
600	15	126	1800	42	120
800	21	135	2000	44	110
1000	26	142	2200	43	94
1200	31	136			

10. Construct a temperature curve from the following figures which were the temperatures of a piece of tool steel placed in a furnace brought to a temperature of 1600° and taken out and allowed to cool. Temperatures were taken after the first minute and every minute thereafter for 20 minutes. 400, 550, 730, 910, 1075, 1240, 1350, 1400, 1410, 1475, 1580, 1600, 1510, 1360, 1280, 1260, 1120, 975, 700 and 450.

Strength of Materials

A **Stress** is a force which acts in the interior of a body, and resists the external forces which tend to change its shape.

The **Unit Stress** is the stress per unit of area, usually per square inch. It is equal to the total stress divided by the area of the cross section in square inches.

Formula: $p = \dfrac{P}{A}$ or $P = A \times p$. (I).

Example 1: Find the unit stress in a bar of iron 2″ square under a load of 3200 lbs.

Formula (I):

$$p = \frac{P}{A} = \frac{3200}{2 \times 2} = \frac{3200}{4} = 800 \text{ lbs. per sq. in. (Ans.).}$$

Example 2: Find the total stress in pounds produced on a 6″ × 6″ sq. oak timber, which is under a unit stress of 250 lbs. per square inch.

Formula (I):

$$P = A \times p = (6 \times 6) \times 250 = 36 \times 250 = 9000 \text{ lbs. (Ans.)}.$$

A **Strain** is the deformation or alternative produced on a body by external forces. When external forces act on a body they produce stresses within the body.

Stresses are of five kinds: **Tensile, Compressive, Transverse** or **Bending, Shearing** and **Torsional.** In most cases a combination of these stresses is produced. Any stress (however small) in a body, produces a deformation or alternation in the shape of the body.

If the stress is not too large, the body will return to its original shape and size when the external force is removed. This property of a body which enables it to return to its original size and shape, is called its **Elasticity.**

If the force be great enough the body will not return to its original size or shape when released, but assumes a new shape, which is called a set, and we say, "its **Elastic Limit** has been exceeded."

Up to the elastic limit the deformation in a body is directly proportional to the load, and the elastic limit may be defined as that point beyond which the deformation ceases to be proportional to the load, or the point at which the rate of stretch begins to increase.

The **Modulus** (or coefficient) **of Elasticity** is the relation between the amount of extension or compression of a body, and the force which reduces that extension or compression. It may be defined as the load per unit of section, divided by the extension per unit of length, and is the quotient obtained by dividing the stress per square inch by the elongation in one inch caused by this stress.

Formula: $E = \dfrac{PL}{Ae}$ or $e = \dfrac{PL}{AE}$. (II).

Example 3: Find the modulus of elasticity of a 2″ sq. steel bar 10 ft. long which elongates 0.080″ under a load of 80,000 lbs.

Formula (II): $E = \dfrac{PL}{Ae}$.

$P = 80{,}000$ lbs.
$L = 10 \times 12 = 120$ inches.
$A = 2 \times 2 = 4$ sq. in.
$e = 0.080''$.

$$E = \frac{80{,}000 \times 120}{4 \times 0.080} = 30{,}000{,}000 \text{ (Ans.)}.$$

Example 4: How much will a bar of cast iron 3″ sq. and 20″ long be compressed under a load of 80,000 lbs.

Formula (II): $e = \dfrac{PL}{AE}$.

$P = 80{,}000$ lbs.
$L = 20$ in.
$A = 3 \times 3 = 9$ sq. in.
$E = 15{,}000{,}000$. (Taken from table III, page 184.)

$$e = \frac{80{,}000 \times 20}{9 \times 15{,}000{,}000} = 0.0118'' \text{ (Ans.)}.$$

Example 5: Find the elongation of a 1″ sq. steel bar 6′ 0″ long under a load of 100,000 lbs.

Formula (II): $e = \dfrac{PL}{AE}$.

$P = 30{,}000$ lbs. per sq. in.
$L = 6 \times 12 = 72''$.
$A = 1$ sq. in.
$E = 30{,}000{,}000$. (Taken from table III, page 184.)

$$e = \frac{30{,}000 \times 72}{1 \times 30{,}000{,}000 \times 1} = 0.072'' \text{ (Ans.)}.$$

The **Breaking** or **Ultimate Stress** of any material is the stress or force which will rupture it.

The ultimate stress in tension, compression and shear for various materials is given in table II, page 183.

Formula (III): $P = A \times S$.

Example 6: What will be the force in pounds necessary to rupture a steel test bar in tension which is 0.505″ in diameter?

Formula (III): $P = A \times S$.

$A = 0.200$ sq. in.

$S = 100{,}000$ lbs. per sq. in. (Taken from table II, page 183.)

$P = 0.200 \times 100{,}000 = 20{,}000$ lbs. (Ans.).

Example 7: What force in pounds will be necessary to punch a 1″ hole in a steel plate $\frac{1}{4}$″ thick?

Formula (III): $P = A \times S$.

$A = 1 \times 3.1416 \times \frac{1}{4} = 0.7854$ sq. in.

$S = 70{,}000$ lbs. (Taken from table II, page 183.)

$P = 0.7854 \times 70{,}000 = 54{,}978$ lbs. (Ans.).

Example 8: What will be the weight in tons necessary to crush a brick pier 10″ square.

Formula (III): $P = A \times S$.

$A = 10 \times 10 = 100$ sq. in.

$S = 2500$ lbs. (Taken from table II, page 183.)

$P = 100 \times 2500 = 250{,}000 \div 2000 = 125$ tons (Ans.).

The **Working Stress** is the stress or load which a body will easily sustain or it is equal to the ultimate stress divided by the factor of safety.

Formula (IV): **Working Stress** $= \dfrac{S}{f}$.

The **Factor of Safety** is the number by which the breaking stress must be divided to obtain the working stress. The

factor of safety in ordinary practice varies from 5 to 30. This depends somewhat upon the formula used, workmanship, character of the load etc. (see table I, page 183).

Example 9: Find the working stress in a beam which has an ultimate strength of 150,000 lbs. per sq. in., if the factor of safety used is 15.

Formula (IV): **Working Stress** $= \frac{S}{f}$.

$$S = 150{,}000 \text{ lbs.}$$
$$f = 15.$$

Working Stress $= \frac{150{,}000}{15} = 10{,}000$ lbs. per sq. in. (Ans.).

For tension, compression (where length does not exceed 10 times its least diameter) and shear

$$P = \frac{AS}{f} \text{ or } A = \frac{Pf}{S}. \quad \text{Formula (V).}$$

Note.—When the length of a bar under compression is longer than 10 times its least diameter, it must be treated as a column and formula No. (VI) must be used.

Example 10: Find the total stress in pounds imposed upon a cast iron bar 3″ square, in tension, if the load is steady.

Formula (V): $P = \frac{AS}{f}$.

$A = 3 \times 3 = 9$ sq. in.

$S = 20{,}000$ lbs. per sq. in. = ultimate strength of cast iron under tension. (Taken from table II, page 183.)

$f = 6$ for cast iron under a steady load. (Taken from table I, page 183.)

$$P = \frac{9 \times 20{,}000}{6} = 30{,}000 \text{ lbs. (Ans.).}$$

Example 11: What will be the safe load to apply under tension on a wrought iron bar 2″ sq. under a varying stress?

Formula (V): $P = \dfrac{AS}{f}$.

$A = 2 \times 2 = 4$ sq. in.
$S = 50{,}000$ lbs. (Taken from table II, page 183.)
$f = 6$. (Taken from table I, page 183.)

$$P = \frac{4 \times 50{,}000}{6} = 33{,}333\tfrac{1}{3} \text{ lbs. (Ans.).}$$

Example 12: Find the size of square cast iron bar necessary to use to resist the compressive stress under a load of $7\frac{1}{2}$ tons?

Formula (V): $A = \dfrac{Pf}{S}$.

$P = 7.5 \times 2000 = 15{,}000$ lbs.
$f = 6$. (Taken from table I, page 183.)
$S = 90{,}000$ lbs. (Taken from table II, page 183.)

$$A = \frac{15{,}000 \times 6}{90{,}000} = 1 \text{ sq. in.} \quad \text{Therefore each side}$$

$$= \sqrt{1''} = 1'' \text{ (Ans.).}$$

Example 13: What should be the diameter of a wrought iron bar to suspend a weight of 50,000 lbs.?

Formula (V): $A = \dfrac{Pf}{S}$.

$P = 50{,}000$ lbs.
$f = 4$. (Taken from table I, page 183.)
$S = 50{,}000$ lbs. per sq. in. (Taken from table II, page 183.)

$$A = \frac{50{,}000 \times 4}{50{,}000} = 4 \text{ sq. in. or } 2.257'' \text{ diameter (Ans.).}$$

Example 14: What should be the diameter of a steel journal in a machine to resist a shearing load of 10,000 lbs. safely?

Formula (V): $A = \dfrac{Pf}{S}$.

P = 10,000 lbs.
f = 15. (Taken from table I, page 183.)
S = 70,000 lbs. (Taken from table II, page 183.)

$$A = \frac{10{,}000 \times 15}{70{,}000} = 2.142 \text{ sq. in. or approximately}$$

1 21/32″ diameter (Ans.).

To find the breaking strength of a column, *i.e.*, a vertical member whose height is greater than 10 times its least diameter the following formula is used.

Formula (VI): $P = \dfrac{SA}{1 + q\dfrac{L^2}{G^2}}$.

Example (15): Find the breaking load of a round wrought iron bar 2″ in diameter and 3 ft. long, under compression, if both ends are rounded.

Formula (VI): $P = \dfrac{SA}{1 + q\dfrac{L^2}{G^2}}$.

S = 50,000 lbs. per sq. in. (Taken from table II, page 183.)
$A = 2 \times 2 \times 0.7854$ sq. in. $= 3.1416$ sq. in.
$q = \dfrac{4}{36{,}000}$. (Taken from table VI, page 186.)
$G^2 = \dfrac{d^2}{16}$ or $\dfrac{2^2}{16}$ or $\dfrac{4}{16}$. (Taken from table V, page 185.)
$L = 3 \times 12 = 36$ inches.

$$P = \frac{50{,}000 \times 3.1416}{1 + \dfrac{4}{36{,}000} \times \dfrac{36^2}{\frac{4}{16}}} = 99{,}670.05 \text{ lbs. (Ans.).}$$

In finding the breaking strength of a beam, *i.e.*, a bar in a horizontal position the bending moments must be taken into consideration, and the following formula may be used:

Formula (VII): $M = S \times R$.

In practice to find the safe working strength of a beam the proper factor of safety must be used which transforms Formula (VII) into $M = \frac{S}{f} \times R$. Formula (VIII).

Example 16: Find the maximum load a 3″ square wooden cantilever 2 ft. long will support at its free end.

$$\text{Bending moment} = \frac{S \times R}{f} = W \times L.$$

(Taken from table IV, page 184.)

$$\therefore\ W = \frac{S \times R}{f \times L} \text{ Formula (IX).}$$

$S = 9000$ lbs. (Taken from table II, page 183.)

$R = \frac{d^3}{6} = \frac{3^3}{6} = \frac{27}{6}$ or $4\frac{1}{2}$. (Taken from table VI, page 186.)

$f = 8$. (Taken from table I, page 183.)

$L = 2 \times 12 = 24$ inches.

$$W = \frac{9000 \times 4\frac{1}{2}}{8 \times 24} = \frac{40{,}500}{192} = 210.9 \text{ lbs. (Ans.).}$$

Example 17: Find the maximum bending moment produced on a simple beam 10 ft. long loaded at the center with a 800 lb. weight.

Formula: $\frac{W \times L}{L}$. (Taken from table IV, page 184.)

$W = 800$ lbs.

$L = 10 \times 12 = 120$ inches.

$$\text{Maximum bending moment} = \frac{800 \times 120}{4}$$

$$= 24{,}000 \text{ inch lbs. (Ans.).}$$

The **Deflection of a Beam** (f) at the critical point or maximum deflection can be found from table IV according to the kind of beam and manner of loading.

Example 18: What is the deflection of a rectangular steel beam fixed at both ends, 5 ft. long, 2″ wide, 3″ deep, supporting a 20,000 pound load in the center?

Formula (X): $S = \dfrac{WL^3}{192E \times I}$. (Taken from table IV, page 184.)

$W = 20{,}000$ lbs.

$L = 5 \times 12 = 60''$.

$E = 30{,}000{,}000$. (Taken from table III, page 184.)

$I = \dfrac{bd^3}{12} = \dfrac{2 \times 27}{12} = 4\frac{1}{2}$. (Taken from table V, page 185.)

$$S = \frac{20{,}000 \times 60^3}{192 \times 30{,}000{,}000 \times 4\frac{1}{2}} = 0.0833'' \text{ (Ans.)}.$$

TABLE I

Factor of Safety "f"

Material	Steady Stress	Varying Stress	Shocks (Machines)
Cast iron	6	15	20
Wrought iron	4	6	10
Steel	5	7	15
Wood	8	10	15
Brick and stone	15	25	30

TABLE II

Ultimate Strengths "S"

Material	Tension	Compression	Shear	Bending
Cast iron	20000	90000	20000	36000
Wrought iron	50000	50000	47000	50000
Steel	100000	150000	70000	120000
Wood	10000	8000	600 to 3000	9000
Stone		6000		2000
Brick	200	2500		

TABLE III

Material	Coefficient of Elasticity (E)	Elastic Limit for Tension
Cast iron	15000000	6000
Wrought iron	25000000	25000
Steel	30000000	50000
Wood	1500000	3000

TABLE IV

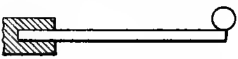
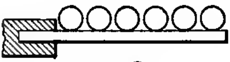
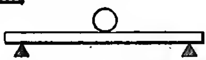
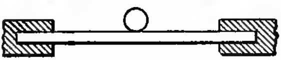
Kind of Beam and Manner of Loading		Bending Moment (M)	Deflection (S)
	Cantilever, load at end	$W \times L$	$\dfrac{W \times L^3}{3E \times I}$
	Cantilever, uniformly loaded	$\dfrac{W \times L}{2}$	$\dfrac{W \times L^3}{8E \times I}$
	Simple beam, load at center	$\dfrac{W \times L}{4}$	$\dfrac{W \times L}{48E \times I}$
	Simple beam, loaded uniformly	$\dfrac{W \times L}{8}$	$\dfrac{5W \times L^3}{384E \times I}$
	Beam fixed at both ends, load at center	$\dfrac{W \times L}{8}$	$\dfrac{W \times L^3}{192E \times I}$
	Beam fixed at both ends, uniformly loaded	$\dfrac{W \times L}{12}$	$\dfrac{W \times L^3}{384E \times I}$

TABLE V

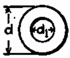
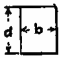
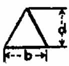
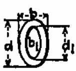
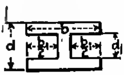
Name and Shape		(I)	(R)	(G^2)
Solid Circle		$\frac{\pi d^4}{64}$	$\frac{\pi d^3}{32}$	$\frac{d^2}{16}$
Hollow Circle		$\frac{\pi(d^4 - d_1^4)}{64}$	$\frac{\pi(d^4 - d_1^4)}{32d}$	$\frac{d^2 + d_1^2}{16}$
Solid Square		$\frac{d^4}{12}$	$\frac{d^3}{6}$	$\frac{d^2}{12}$
Hollow Square		$\frac{d^4 - d_1^4}{12}$	$\frac{d^4 - d_1^4}{6d}$	$\sqrt{\frac{d^2 + d_1^2}{12}}$
Solid Rectangle		$\frac{bd^3}{12}$	$\frac{bd^2}{6}$	$\frac{b^2}{12}$
Hollow Rectangle		$\frac{bd^3 - b_1d_1^3}{12}$	$\frac{bd^3 - b_1d_1^3}{6d}$	$\sqrt{\frac{bd^3 - b_1d_1^3}{12(bd - b_1d_1)}}$
Solid Triangle		$\frac{bd^3}{36}$	$\frac{bd^2}{24}$	$\frac{d^2}{18}$
Solid Ellipse		$\frac{\pi bd^3}{64}$	$\frac{\pi bd^2}{32}$	$\frac{b^2}{16}$
Hollow Ellipse		$\frac{\pi(bd^3 - b_1d_1^3)}{64}$	$\frac{\pi(bd^3 - b_1d_1^3)}{32d}$	$\frac{b^3d - b_1^3d_1}{16(bd - b_1d_1)}$
I Beam		$\frac{bd^3 - b_1d_1^3}{12}$	$\frac{bd^3 - b_1d_1^3}{6d}$	$\frac{b^3d - b_1^3d_1}{12(bd - b_1d_1)}$
Even Cross		$\frac{\text{Area} \times d^2}{19}$	$\frac{\text{Area} \times d}{9.5}$	$\frac{d^2}{22.5}$
Even Angle		$\frac{\text{Area} \times d^2}{10.2}$	$\frac{\text{Area} \times d}{7.2}$	$\frac{d^2}{25}$

TABLE VI

Material	Both Ends Flat or Fixed	One End Round	Both Ends Round
Cast iron	$\frac{1}{5000}$	$\frac{1.78}{5000}$	$\frac{4}{5000}$
Wrought iron	$\frac{1}{36000}$	$\frac{1.78}{36000}$	$\frac{4}{36000}$
Steel	$\frac{1}{25000}$	$\frac{1.78}{25000}$	$\frac{4}{25000}$
Wood	$\frac{1}{3000}$	$\frac{1.78}{3000}$	$\frac{4}{3000}$

If the weight of the beam is large in comparison to the load on the beam, then it must be considered as a load uniformly distributed over the whole length of the beam (provided the beam is of uniform cross-section throughout).

A **Simple Beam** is one that is merely supported at the ends.

In the expression for (R) (table V) "d" is always understood to be the vertical side or depth.

The moment of inertia (I) (table V) is taken about a plane perpendicular to "d" and lying in the same plane.

EXERCISES

1. A square cast iron pillar 18″ long is required to sustain a steady load of 75000 lbs. What must be the length of a side?

2. What would be the elongation of a round wrought iron bar 24″ long and $1\frac{1}{2}$″ in diameter under a tensile stress of 15 tons?

3. What is the breaking load of a cast iron simple beam, uniformly loaded, which is 20 ft. long between supports, and of hollow rectangular cross section 6″ × 8″ outside and 4″ × 6″ inside, neglecting its own weight?

4. What is the deflection of a solid wrought iron beam 6″ wide and $2\frac{3}{4}$″ deep, fixed at both ends, 7 ft. between supports, with a load of 21000 lbs. in the center?

5. What is the depth "d" of a steel cantilever 36″ in length, that will sustain a weight of 4000 lbs. at the end, if beam is rectangular and $2\frac{1}{2}$″ wide, neglecting its own weight?

6. What is the breaking load on an elliptical wooden column, having rounded ends, the diameters of the cross section being 12″ and 8″ and the column 18 ft. long?

7. A wrought iron bar is to support in tension a load of 20 tons, variable. Find the diameter of the bar required.

8. What would be the elongation of the bar in problem No. 7 if the length was 5 ft.?

9. A square bar firmly held at one end, is supporting a load of 3000 lbs. at the outer free end. If the load is steady, the bar made of structural steel and is $2\frac{1}{2}$ ft. long, what size should it be?

10. If an elongation of 0.015″ is produced in a steel bar 10″ long, 2″ square by a tensile load of 90 tons, what is the modulus (or coefficient) of elasticity?

Springs

Springs are divided into three general classes: elliptical, helical and spiral.

A **Spiral Spring** is one which is wound around a fixed point or axle, the diameter of each coil gradually increasing. (Fig. I.)

A **Helical Spring** is one which is wound around an arbor, and at the same time advancing like the thread of a screw.

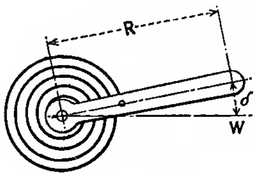


FIG. I. SPIRAL SPRING

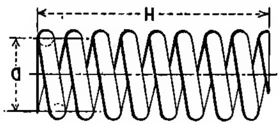
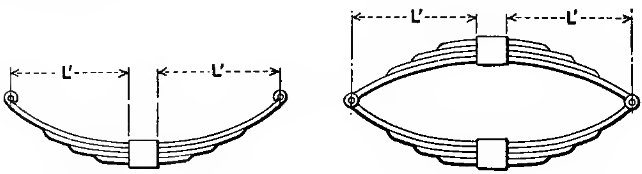


FIG. II

(Fig. II.) A good proportion for this kind of spring is to make the mean diameter from 5 to 10 times the diameter of wire used.



HALF ELLIPTICAL FULL ELLIPTICAL

FIG. III

An **Elliptical or Laminated Spring** is made up of flat bars, plates or leaves of uniformly varying length, placed one upon the other and held together by bolts and clips. (Fig. III.)

Abbreviations Used in Formulas

W = safe load in lbs. per sq. in., or maximum carrying capacity.

W' = load which causes the deflection δ' (where W' is less than W) in lbs. per sq. in.

δ = total deflection in inches.

δ' = total deflection in inches under the load W'.

f = safe strength in lbs. per sq. in. (80,000 lbs. per sq. in. for ordinary spring steel).

b = width of spring in inches.

t = thickness of spring in inches.

d = diameter of spring wire in inches.

D = mean diameter of helical spring in inches.

H = free height of helical spring in inches (height without load).

h = solid height of helical spring in inches (height when completely compressed).

L = effective span = $2L'$.

l = length of spring in inches.

n = no. of leaves in half elliptical spring, or $\frac{1}{2}$ of total no. of leaves in full elliptical spring.

R = length of load arm in inches.

E = modulus of elasticity for tension and compression = (30,000,000 for steel).

G = modulus of elasticity for torsion = (12,600,000 for steel).

r = ratio of no. of full length leaves to total no. of leaves.

Spiral Springs:

$$W = \frac{fd^3}{10R}, \qquad \delta = \frac{20WlR^2}{Ed^4} = \frac{2flR}{Ed},$$

for round spring stock.

$$W = \frac{fbt^2}{6R}, \qquad \delta = \frac{12WlR^2}{Ebt^3} = \frac{2flR}{Et},$$

for rectangular spring stock.

Example: Find the safe load for spiral spring 2″ wide $\frac{1}{4}$″ thick, 6′–0″ long, when the load arm is 10″. Also find the deflection δ under the load.

$$W = \frac{fbt^2}{6 \times R} = \frac{80{,}000 \times 2 \times (\frac{1}{4})^2}{6 \times 10} = 166.7 \text{ lbs. (Ans.).}$$

$$\delta = \frac{2flR}{Et} = \frac{2 \times 80{,}000 \times 6 \times 12 \times 10}{30{,}000{,}000 \times \frac{1}{4}} = 15.36'' \quad \text{(Ans.).}$$

Helical Springs: (*a*) round bar stock.

$$W = \frac{G\delta d^5}{8hD^3} = \frac{\pi f d^3}{8D},$$

$$\delta = H - h = \frac{\pi f h D^2}{Gd^2}. \qquad \delta' = \frac{8W'hD^3}{Gd^5},$$

$$H = h + F = h + \frac{\pi f h}{G}\left(\frac{D}{d}\right)^2,$$

$$l = \frac{\pi h D}{d} = \frac{H}{\dfrac{fD}{Gd} + 0.32\dfrac{D}{d}}.$$

(*b*) rectangular bar stock.

$$W = \frac{fbt\sqrt{t^2 + b^2}}{3D} = \frac{G\delta bt(t^2 + b^2)}{9.4hD^3},$$

$$\delta = \frac{\pi f h D^2}{Gt\sqrt{t^2 + b^2}} = \frac{H}{1 + \dfrac{Gt\sqrt{t^2 + b^2}}{\pi f D^2}},$$

$$\delta' = \frac{9.4hD^3W'}{Gbt^2(t^2 + b^2)},$$

$$H = h + \frac{\pi h f D^2}{Gt\sqrt{t^2 + b^2}},$$

$$l = \frac{\pi D h}{t} = \frac{H}{\dfrac{fD}{G\sqrt{t^2+b^2}} + 0.32\dfrac{t}{D}}.$$

Example: Find the diameter and length of wire for a round helical spring, mean diameter 1″, solid height 1″, for a load of 1000 lbs.

$$W = \frac{\pi f d^3}{8D},\quad d = \sqrt[3]{\frac{8WD}{\pi f}} = \sqrt[3]{\frac{8 \times 1000 \times 1}{3.14 \times 80000}} = 0.179'' \quad \text{(Ans.)}.$$

$$l = \frac{\pi h D}{d} = \frac{3.14 \times 1 \times 1}{0.179} = 17.49'' \text{ (Ans.)}.$$

Elliptical Springs:

$$W = \frac{2}{3}\frac{fnbt^2}{L}.$$

$$\delta = \frac{1}{2}\frac{fL^2}{Et} \text{ for full elliptic.}$$

$$\delta = \frac{1}{2+r}\frac{fL^2}{Et} \text{ for full elliptic with more than one full leaf.}$$

$$\delta = \frac{1}{4}\frac{fL^2}{Et} \text{ for half elliptic.}$$

$$\delta = \frac{1}{2(2+r)}\frac{fL^2}{Et} \text{ for half elliptic with more than one full leaf.}$$

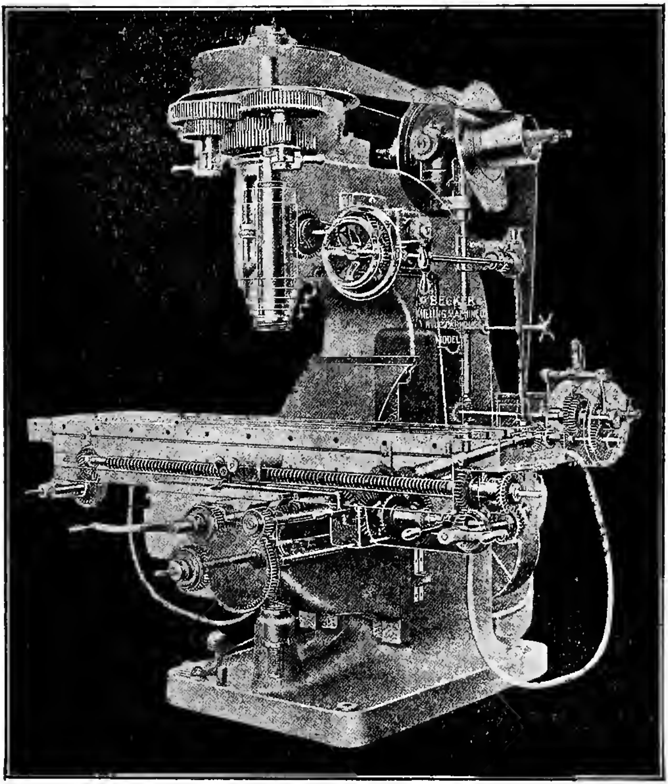
Example: Find the safe carrying load and maximum deflection of a semi-elliptical spring 2″ wide, 0.266″ thick, 9 leaves and 42″ span, taking the value of f = 40,000 lbs. per sq. in.

$$W = \frac{2}{3}\frac{fnbt^2}{L} = \frac{2}{3} \times \frac{40{,}000 \times 9 \times 2 \times (0.266)^2}{42}$$

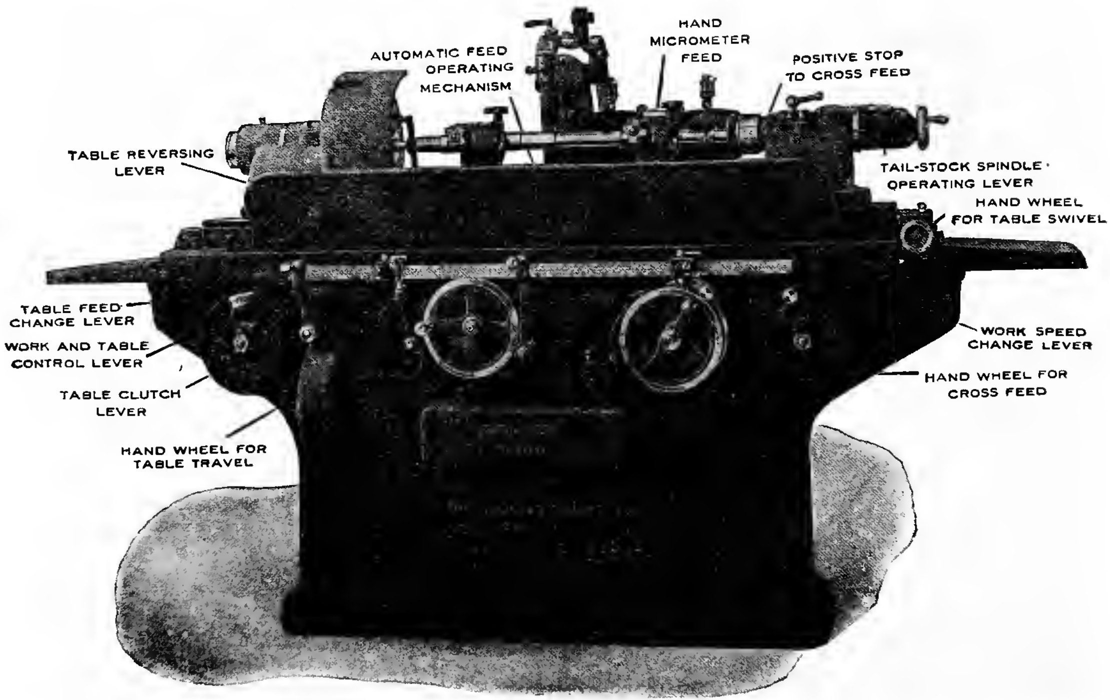
$$= 808 \text{ lbs. (Ans.)}.$$

$$\delta = \frac{1}{4}\frac{fL^2}{Et} = \frac{1}{4} \times 40{,}000 \times \frac{(42)^2}{30{,}000{,}000 \times 0.266}$$

$$= 2.409'' \text{ (Ans.)}.$$



VERTICAL MILLING MACHINE



UNIVERSAL GRINDING MACHINE

EXERCISES

1. What is the deflection of a spiral spring 1″ wide, $\frac{1}{8}$″ thick and 2′–0″ long, under the maximum safe load which has a lever arm of 3″?

2. What is the maximum carrying capacity of a $\frac{1}{4}$″ round steel helical spring with a mean coil diameter of 1″?

3. If the solid height of the above spring is $2\frac{1}{2}$″, what is the free height?

4. Also find the deflection of the above spring under a load of 250 lbs.

5. Find the diameter of wire required for a round helical spring under the following conditions:

Maximum load. . . . . .1100 lbs.
Mean diameter of coil. . . . .$1\frac{1}{2}$″

6. Find the total deflection of the above spring under a load of 600 lbs. when the solid height is 4″.

7. A helical spring of $\frac{1}{2}$″ steel wire is to have a compression of $1\frac{1}{2}$″ under a load of 2000 lbs. What is the outside diameter and solid height of the coil?

8. Also find the total length of the above spring.

9. A semi-elliptical spring consists of 7 plates of $\frac{3}{8}$″ thickness, $3\frac{1}{2}$″ wide, with a 30″ span. Find the safe load and deflection, taking the value of $f = 80{,}000$ per sq. in. $E = 25{,}000{,}000$.

10. A semi-elliptical spring has 11 leaves of 9/32″ thickness, 2″ wide, with a span of 52″. Find the safe load using $f = 40{,}000$ lbs. per sq. in. $E = 30{,}000{,}000$.

Pipes and Cylinders

When a hemispherical vessel, suspended by a string as in Fig. I under internal steam pressure, will move neither to the right nor to the left, this proves that the total pressure on the curved surface in direction F is equal to that upon the flat surface in direction E.

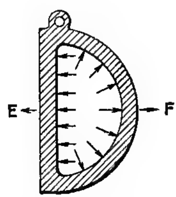


FIG. I

The flat surface is called the **Projected Area** of the curved surface.

Abbreviations Used in Formulas for Cylinders

d = inside diameter of cylinder in inches.
t = thickness of cylinder wall in inches.
p = pressure in lbs. per sq. in.
s = working stress of cylinder walls in lbs. per sq. in.
l = length of cylinder in inches.
P = total pressure in one direction in lbs. per sq. in.

Total pressure (P) is equal to (pdl).

The resistance of the boiler to this pressure on each side A and B (Fig. II) is equal to (stl).

Therefore: $pld = 2stl.$

$$pd = 2st.$$

Formula 1: $p = \dfrac{2st}{d}$ or $t = \dfrac{pd}{2s}$.

The pressure has also a tendency to rupture the receptacle circumferentially by pulling it apart lengthwise. (See Fig. III.)

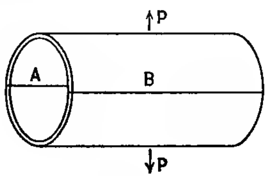


FIG. II

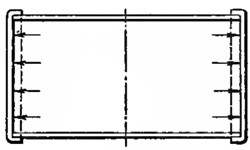


FIG. III

The area of cylinder head against which the pressure acts is equal to $\frac{1}{4}\pi d^2$ and the total load exerted on the shell circumferentially is equal to the total area in sq. in. of one end, times pressure per sq. in.

The area of metal to resist this pressure is πdt. The total resistance is πdts. Therefore $\frac{1}{4}\pi d^2 p = \pi dts$.

Formula 2: $p = \dfrac{4ts}{d}$ or $t = \dfrac{pd}{4s}$.

The thickness of cylinder obtained by the formula (2) is just half of that of formula (1). Thus, the cylinder generally fails along the longitudinal section.

Example: Find the thickness of a cast iron cylinder 4″ in diameter to resist a 400 lb. pressure, taking ultimate tensile strength of C.I. as 20,000 lbs. per sq. in., using a factor of safety of 10.

$$t = \frac{pd}{2s} = \frac{400 \times 4}{\dfrac{2 \times 20{,}000}{10}} = 0.4'' \text{ (Ans.).}$$

Where boiler flues, tubes, etc. are under external pressure, this pressure has a tendency to change the shape of the round tubes into an elliptical cross-section. This distorsion when once begun, increases rapidly and failure occurs by the collapsing of the tube.

Abbreviations Used in Formulas for Pipes and Tubes

p = collapsing pressure in lbs. per sq. in.
t = thickness of tubes in inches.
d = outside diameter of tube in inches.
l = length of pipe in inches.

Formula 3: $p = \left(87000\,\frac{t}{d}\right) - 1400$. (For lap welded steel tube when p is greater than 600 lbs. per sq. in.)

Formula 4: $p = 50000000\left(\frac{t}{d}\right)^3$. (For lap welded steel tube when p is less than 600 lbs. per sq. in.)

EXERCISES

1. Find the thickness of steam cylinders 12″ in diameter to resist the steam pressure of 120 lbs. per sq. in., using working strength of 2000 lbs. per sq. in.

2. Find the maximum steam pressure allowable for a cast iron cylinder 6″ in diameter $\frac{3}{16}$″ thick, taking working strength as 2000 lbs. per sq. in.

3. Find the thickness of a cast iron water pipe 6″ in diameter, under a pressure of 300 lbs. per sq. in., taking working strength as 1500 lbs. per sq. in.

4. Find the thickness of a gas engine cylinder 3″ in diameter, assuming the maximum explosion pressure to be 400 lbs. per sq. in. (Use the formula No. 2—f = 2000 lbs. per sq. in.)

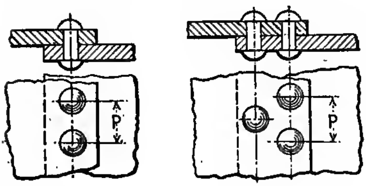
5. What is the collapsing pressure of a lap welded steel tube $\frac{1}{16}$″ thick and 2″ in diameter?

Riveted Joints

Riveted Joints may be classified according to the method of connecting the plate and the number of rows of rivets used.

Lap Joints are joints where main plates overlap each other, as in Fig. I (*a* and *b*).

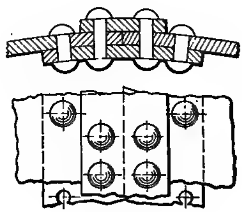
Butt Joints are joints where edges of main plate butt against each other and the connection is made through cover plates, as in Fig. II.



(*a*) Single riveted lap joint

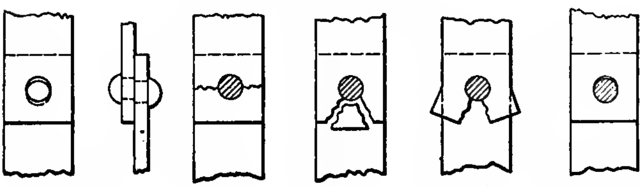
(*b*) Double riveted lap joint

FIG. I



Butt joint
FIG. II

Several methods of failure



(1) Shearing (2) Tearing (3) Tearing (4) Bending (5) Bearing

(1) Failure due to shearing stress in the rivet.
(2) Failure due to tensile stress of plate.
(3) Failure due to shearing stress of plate.
(4) Failure due to bending stress in the plate.
(5) Failure due to bearing or compressive stress in the rivet.

Abbreviations Used in Formulas

f_t = tensile strength of plate in lbs. per sq. in.
f_s = shearing strength of rivets in lbs. per sq. in.
f_c = compressive strength of rivets in lbs. per sq. in.
t = thickness of plate in inches.
d = diameter of rivet in inches.
P = pitch of rivets in inches. (Distance from center of one rivet to another.)

Formula for single riveted lap joint (Fig. I–*a*):

Resistance to shearing one rivet $= \frac{\pi d^2}{4} f_s$.

Resistance to tearing between rivets $= (P-d)\, t.f_t$.

Resistance to compressing rivet or plate $= d.t.f_c$.

$$\frac{\pi d^2}{4} f_s = d.t.f_c. \qquad \text{or} \qquad d = 1.27 \frac{f_c t}{f_s}.$$

$$\frac{\pi d^2}{4} f_s = (P-d)\, t.f_t. \qquad \text{or} \qquad P = \frac{\pi d^2}{4} \frac{f_s}{tf_t} + d.$$

Formula for double riveted lap joints (Fig. I–*b*):

Resistance to shearing 2 rivets $= \frac{2\pi d^2}{4} f_s$.

Resistance to tearing between 2 rivets $= (P-d)\, t.f_t$.

Resistance to compressing in front of 2 rivets $= 2d.t.f_c$.

$$\frac{2\pi d^2}{4} f_s = 2d.t.f_c., \qquad d = 1.27 \frac{f_c t}{f_s},$$

$$\frac{2\pi d^2}{4} f_s = (P-d)\, t.f_t., \qquad p = \frac{\pi d^2}{2} \frac{f_s}{tf_t} + d.$$

The efficiency of a rivet joint is equal to the ratio of the section of plate left between the rivets to the sections of solid plate; or the ratio of the clear distance between 2 adjacent rivet holes to the pitch.

Material	Tensile Strength in Lbs. (f_t)	Compressive Strength in Lbs. (f_c)
Wrought iron	50,000	80,000
Steel	56,000	90,000
	Shearing Strength in Lbs. (f_s)	
Wrought iron	40,000	
Steel	45,000	

EXERCISES

1. Calculate the diameter and pitch of steel rivets, single riveted lap joint, for $\frac{5}{16}''$ steel plates.

2. Find the diameter of rivets if the above plates are wrought iron.

3. Find the diameter and pitch of steel rivet for $\frac{1}{2}''$ wrought iron plate, double riveted lap joint.

4. Find the efficiency of joints of the above problem.

5. Determine the required number of steel rivets for a joint to carry 20,000 lbs., using $\frac{3}{8}''$ steel plate.

6. If 2 plates 4″ wide and $\frac{3}{8}''$ thick are connected by two $\frac{7}{8}''$ rivets, what load will the joint safely carry?

Logarithms

Logarithms are used to facilitate mathematical calculations involving multiplication, division, involution and evolution, which would otherwise require considerable labor and consequent risk of error.

When a number (N) is equal to the (X) power of a number (a), or $N = a^x$, X is called the logarithm of the number (N), and (a) is called its **Base.** It is expressed in the following way: $\log_a N = X$.

Rule 1.—The logarithm of the product of two or more numbers is equal to the sum of the logarithms of several factors. Example: $\log_a (M \times N) = \log_a M + \log_a N$.

Rule 2.—The logarithm of the quotient of two numbers is equal to the remainder found by subtracting the logarithm

of the divisor from the logarithm of the dividend. Example: $\log_a M/N = \log_a M - \log_a N$.

Rule 3.—The logarithm of the power of a number is the product of the logarithm of the number multiplied by the exponent or index of the power. Example: $\log_a N^k = K \log_a N$.

Rule 4.—The logarithm of the root of a number is the quotient found by dividing the logarithm of the number by the index of the root. Example: $\log_a \sqrt{N} = 1/k \log_a N$.

A logarithm consists of two parts, the whole number called the **Characteristic** and the decimal the **Mantissa.** Example: Log 1.2549. The figure (1) is the characteristic and the figure (2549) is the mantissa.

When the base (a) of logarithms is 10, logarithms are called **Common Logarithms.**

When the base (a) of logarithms is (e) or 2.7182819+, logarithms are called **Natural, Hyperbolic** or **Napierian Logarithms.**

The common logarithm is used in ordinary calculations, and the hyperbolic logarithm is used in higher mathematics.

In this book we treat only with common logarithms, as

$$10^{.3010} = 2 \quad \text{or} \quad \log 2 = 0.3010,$$
$$10^{1.3010} = 20 \quad \text{or} \quad \log 20 = 1.3010,$$
$$10^{2.3010} = 200 \quad \text{or} \quad \log 200 = 2.3010.$$

The above figures show that the logarithms of 2, 20 and 200 have the same mantissa, the only difference being the value of characteristics. The mantissa of any number from 1 to 1000 can be found from the table below. Therefore we must prefix a suitable characteristic to it afterward

$$\text{since } 10^0 = 1 \quad \text{therefore } \log 1 = 0,$$
$$10^1 = 10 \qquad \log 10 = 1,$$
$$10^2 = 100 \qquad \log 100 = 2,$$
$$10^{-1} = 1/10 = 0.1 \qquad \log 1/10 = -1,$$
$$10^{-2} = 1/100 = 0.01 \qquad \log 1/100 = -2.$$

It is evident that the common logarithm of any number between

1	and 10	will be	0 plus a decimal,
10	and 100	will be	1 plus a decimal,
100	and 1000	will be	2 plus a decimal,
1	and 0.1	will be	− 1 plus a decimal,
0.1	and 0.01	will be	− 2 plus a decimal.

The above figures show that the characteristics of all numbers less than 1 have a negative value, and the characteristics of all the numbers greater than 1 have a positive value. The mantissa is always made positive.

When the characteristic has a negative value, it is customary to write the minus sign over the top, or to add 10 to it and to indicate the subtraction of 10 from the resulting logarithm. Thus $\log 0.2 = \bar{1}.3010$ or $9.3010 - 10$.

Example 1: Find the logarithms of 2056, 20.56 and 0.002056.

First locate the number 20 in the first column and find the figure 5 at the top of column, then follow the column downward and find the mantissa of log 205 which equals 0.3118 and of log 206 which equals 0.3139.

The difference between these two mantissas is 21 and the difference between 2060 and 2050 is 10, also between 2056 and 2050 is 6. Hence 6/10 of 21 equals 13− or 13. This must be added to 0.3118. This is the mantissa of log 2056, which equals 0.3131.

Therefore $\log 2056 = 3.3131$ (Ans.).
$\log 20.56 = 1.3131$ (Ans.).
$\log .002056 = 3.3131$ (Ans.).

Example 2: Multiply 20.32 by 0.03849 by 0.0023.

$$\log 20.32 = 1.3079,$$
$$\log 0.03849 = 2.5853,$$
$$\log 0.0023 = 3.3617.$$

From rule 1—log $(20.32 \times 0.03849 \times 0.0023) = 1.3079 + \bar{2}.5853 + \bar{3}.3617 = \bar{3}.2549$.

When adding logarithms together, first add the mantissa and characteristic separately and then add the two together, as this eliminates confusion. That is

$$1 + \bar{2} + \bar{3} = \bar{4}.$$
$$0.3079 + 0.5853 + 0.3617 = 1.2549.$$
$$\bar{4} + 1.2549 = \bar{3}.2549.$$

Next, from the same table we find the number whose mantissa is 0.2549, that is $0.2549 = \log 1.799$.

Therefore $3.2549 = \log 0.001799 = 0.001799$ (Ans.).

Example 3: Calculate the value $(0.023)^{\frac{2}{3}}$ (using rule No. 3).

$$\begin{aligned}
\log 0.023 &= 2.3617 \\
\tfrac{2}{3} \log 0.023 &= \tfrac{2}{3} \times \bar{2}.3617 \\
&= \tfrac{2}{3} \times (\bar{2} - 1 + .3617 + 1) \\
&= \tfrac{2}{3}(\bar{3} + 1.3617) = \bar{2} + 0.9078 \\
&= \bar{2}.9078 \\
&= \log 0.08087 = 0.08087 \text{ (Ans.)}.
\end{aligned}$$

EXERCISES

1. Find the logarithm of 0.049 and 0.49.
2. Find the logarithm of 3257 and 0.3257.
3. Find the number of which the logarithm is 3.2833.
4. Find the number of which the logarithm is 1.9727.
5. Find the number of which the logarithm is 3.5123.
6. Find the number of which the logarithm is 0.6395.
7. Multiply 3.891 by 0.00876.
8. Multiply 2.562 by 0.01035.
9. Multiply $0.0323 \times 0.00452 \times 8890000$.
10. Divide 3320. by 954.
11. Divide 763 by 0.368.
12. Divide 316.6 by 4.21.
13. Calculate $6190 \times 3.25 \div 2625$.
14. Calculate $6542 \div 318 \div 66.82$.
15. Find the square root of 3256. and 359200.
16. Find the cube root of 54.22 and 66.15.

17. Find the square of 8.252 and 786.1.
18. Find the fifth power of 0.2382.
19. Calculate $(8025)^{2/3}$.
20. Calculate $\frac{\sqrt[2]{0.00052}}{\sqrt[5]{0.006814}}$.

Heat

Heat.—The molecules of all matter are always in motion, the rapidity of this motion determines the intensity of heat. Heat is the most common form of energy.

Temperature is the measure of the intensity of molecular motion or of the heat, being registered by thermometers or pyrometers. There are 2 kinds of thermometer scales in general use; the Fahrenheit (F.) and the Centigrade (C.).

The following formulas are used for converting temperatures given in anyone of the scales to the other scale.

$$F.^\circ = (9/5 \times C.^\circ) + 32.$$
$$C.^\circ = 5/9 \times (F.^\circ - 32).$$

The **Freezing Point** of water is 32° Fahrenheit, or 0° Centigrade, and the **Boiling Point** of water is 212° Fahrenheit, or 100° Centigrade, at atmospheric pressure (14.7 lbs.).

Absolute Temperature.—The volume of a perfect gas increases 1/273 of its volume at 0° C. for every increase of temperature of 1° C., and decreases 1/273 of its volume for every decrease of temperature of 1° C. Thus, the volume of the imaginary gas would be reduced to nothing at − 273° C. (or − 492° F.). This point is called the **Absolute zero.** Absolute temperatures are measured either by the Fahrenheit or Centigrade scales, from this zero point. The freezing point corresponds to 492° F., or 273° *F.* absolute.

$$\text{Absolute temperature} = 460^\circ + F.^\circ$$
$$= 273^\circ + C.^\circ$$

Specific Heat of a body is its capacity for heat, or the amount of heat in thermal units required to raise the tem-

perature of 1 lb. of the body through 1 degree F., compared with that required to raise an equal weight of water 1 degree.

If a body with a given temperature is put in a vessel containing a measured weight of water at a certain temperature, the final temperature of the mixture can be found by the following formula.

Heat lost by body = Heat gained by water.

Weight of body × specific heat × fall of temp. = weight of water × specific heat of water or 1 × rise of temp.

Example: Find the specific heat of copper from the following data: $\frac{1}{2}$ lb. of copper is heated to 212° F. and plunged into 20 oz. of water at 60° F., if the resulting temperature was 65.56° F.

$$\frac{20 \times 1 \times (65.56 - 60)}{\frac{16}{2} \times (212 - 65.56)} = 0.095 \text{ (Ans.)}.$$

Water at 39.1° F.	1.00	Steel	0.117
" " 212° F.	1.03	Copper	0.095
Ice " 32° F.	0.504	Charcoal	0.241
Steam " 212° F.	0.481	Air at constant pressure	0.238
Mercury	0.033	Oxygen " "	0.218
Cast iron	0.13	Hydrogen " "	3.409
Wrought iron	0.113	Nitrogen " "	0.244

The specific heat of various substances are given in the following table.

A BRITISH THERMAL UNIT (B.t.u.) is the quantity of heat required to raise the temperature of 1 lb. of pure water 1° F. at or near 60°, at which time water is at its maximum density.

A FRENCH THERMAL UNIT (Calorie) is the quantity of heat required to raise the temperature of 1 kilogram of pure water 1° C. at 15° C.

1 B.t.u. = 0.252 Calorie.

1 Calorie = 3.968 B.t.u.

The **Mechanical Equivalent of Heat** is the number of ft. lbs. of mechanical energy equivalent to 1 B.t.u. and is equal to 778.

1 B.t.u. = 778 ft. lbs.

Latent Heat is the quantity of heat units absorbed or given out, in changing one pound of a substance from one state to another without changing its temperature.

When a body passes from the solid to the liquid state, its temperature remains stationary at a certain melting point during the whole operation of melting and in order to make that operation go on, a quantity of heat must be supplied to the substance. This quantity of heat is called the **Latent Heat of Fusion.**

When a body passes from the liquid to the solid state, its temperature remains stationary during the whole operation of freezing, and a quantity of heat equal to the latent heat of fusion is produced in the body and rejected into the atmosphere or other surrounding substances.

When a body passes from the solid or liquid state to the gaseous state, its temperature during the operation remains stationary at a certain boiling point, depending upon the pressure of the vapor produced, and in order to make the evaporation go on, a quantity of heat must be supplied to the substance evaporated, whose amount for each unit of weight of the substance evaporated depends upon the temperature. This heat is called the **Latent Heat of Evaporation.**

The following table shows the latent heat of various substances.

Latent Heat of Fusion				Latent Heat of Evaporation at Atmospheric Pressure (14.7)		
Material	B.T.U.	Material	B.T.U.		Boiling Point	B.T.U.
Ice	144	Tin	26	Water	212° F.	966
Cast iron (gray)	41	Lead	10	Alcohol	172° F.	364
Cast iron (white)	60	Zinc	51	Ether	95° F.	163

Example: Find the B.t.u. required to change 100 lbs. of water at 100° F. into steam at 212° F.

No. of heat units required to raise the temp. of water from 100° to 212°.	= (212–100) × 100 = 11,200 B.t.u.
No. of heat units required to evaporate the water from and at 212°.	= 966 × 100 = 96,600 B.t.u.

Total heat units = 11,200 + 96,600 = 107,800 B.t.u. (Ans.).

Expansion of gas:

1st law—The volume of a given portion of gas varies inversely as its pressure if the temperature is constant.

$$P = \text{pressure of gas.}$$
$$V = \text{volume of gas.}$$
$$PV = P_1V_1.$$

2d law—The increase in volume of a given portion of gas varies directly as the increase in temperature, if the pressure is constant.

$$\alpha = \text{co-efficient of cubical expansion.}$$
$$V = \text{initial volume.}$$
$$V_1 = \text{increase of volume.}$$
$$V_2 = \text{total volume after increase.}$$
$$t = \text{rise of temp. in degrees.}$$
$$V_1 = V\alpha t.$$
$$V_2 = V_1 + V = V + V\alpha t = V(1 + \alpha t).$$

Isothermal expansion or compression takes place when a gas is expanded or compressed with an addition or transmission of sufficient heat to maintain a constant temperature.

When an expanding gas forces a piston forward against a resistance, it does work requiring expenditure of heat. Such heat being abstracted from the gas, decreases its temperature.

Adiabatic expansion or compression takes place when a gas is expanded or compressed without the transmission of heat

to or from it. For example, a gas will follow the adiabatic curve if a gas could be expanded or compressed in a receptacle of an absolute non-conducting material.

For the adiabatic expansion of a perfect gas, the following formula is used:

$$PV^r = P_1V_1^r.$$

$$\text{where}\begin{cases} r = 1.408 \text{ for air.} \\ r = 1.3 \text{ for superheated steam.} \\ r = 10/9 \text{ for saturated steam.} \\ r = 1.411 \text{ for carbon monoxide.} \end{cases}$$

P = initial pressure.

P_1 = final pressure.

V = initial volume.

V_1 = final volume.

EXERCISES

1. Transfer the following Centigrade reading into Fahrenheit:
(a) 23° (b) 7° (c) −15°

2. Transfer the following Fahrenheit reading into Centigrade:
(a) 63° (b) 15° (c) −4°

3. Change the following temperatures into absolute temperatures:
(a) 212° F. (b) −60° F. (c) 12° C.

4. A vessel containing 200 lbs. of water melts a piece of ice, and the fall of temperature is 15° F. What is the weight of the ice?

5. Equal weight of hot water and ice are placed in one vessel and the temperature of water after the ice is melted is 45° F. What is the initial temperature of hot water?

6. How many lbs. of hot water at 200° F. will be required to warm up a copper plate weighing 40 lbs., from 50° to 150°?

7. How many thermal units will be given out in cooling and freezing 10 lbs. of water at 100° F.?

8. How many gallons of gasoline containing a heating power of 19,000 B.t.u. per lb. will be required in one hour, to develop 10 i.h.p. in a gasoline engine, if the thermal efficiency is 17%? (1 lb. = 0.17 gallon.)

9. What horse power can be developed by using 20 gallons of gasoline per hour, which contains a heating value of 110,000 B.t.u. per gallon, if the loss of heat by jacket water is 50%, by exhaust 17% and by radiation 16%?

10. A 100 h.p. steam engine consumes 25 lbs. of steam per i.h.p. per hour. How many lbs. of coal will be required in one hour, if the feed water temperature is 212° F., and the heating value of coal is 14,000 B.t.u. per lb.?

11. Find the compression pressure of a gasoline engine when the initial pressure is 13 lbs. per sq. in. and the compression ratio is 4 to 1, assuming the expansion of gas follows the adiabatic expansion of superheated steam. The compression ratio is the ratio of the total cylinder volume, *i.e.*, compression volume plus piston displacement, to the compression volume.

Metal Cutting

Steel used for cutting metals are broadly classified into tool steels and high speed steels.

Tool Steels or high carbon steels contain 0.60 to 1.50% of carbon. The percentage of carbon determines the hardness of the cutting tool, *i.e.*, the heat treatment being equal.

High Speed Steels contain several other ingredients such as tungsten, chromium, manganese, silicon, molybdenum, vanadium and nickel. Of all these ingredients, tungsten and chromium are the most important factors as they give the steel the property of red hardness; *i.e.*, the tool does not lose its cutting ability under very high speeds or heat.

Cutting Speeds depend upon the following conditions:

1. Kind of steel to be used, whether tool steel or high speed steel.
2. Shape of tool, whether narrow or broadnosed.
3. Lip angle of tool, or included angle of nose.
4. Sharpness of tool.
5. Position of tool in the tool post.
6. Depth of cut and amount of feed.
7. Material to be cut, whether soft, medium or hard, or whether brass, cast iron or steel.
8. Cooling medium, whether used or not, the amount of cooling and lubricating effect produced.
9. Heat treatment of steel.

10. Elasticity of work or tool, which causes chattering.
11. Rigidness with which work is held.
12. Condition of machine to be used.

The power required to remove a given amount of metal depends upon the shape and sharpness of the cutting tool, hardness of the work, depth and feed of cut, lubrication of cutting point, and also upon the kind and condition of machine.

The **Average Horse Power Required** to drive the machine can be determined by the product of the amount of chips (W) multiplied by 2 constants ($Y + Z$). (Y) varies with the kind of material to be cut and (Z) with the kind of machine to be used.

h.p. = Horse power required to drive the machine.
W = Weight of metal removed per hour in lbs.
Y = Constant 1.0 for cast iron.
1.3 for mild steel.
2.0 for tool steel.
0.7 for bronze.
Z = Constant 0.035 for lathe.
0.030 for shaper.
0.025 for miller.
0.030 for drill.

h.p. = YZW.

Example 1: What horse power will be required to run a lathe at 50 r.p.m. to turn a cast iron wheel 8″ in diameter, with a 1/32″ feed and a 1/8″ depth of cut?

$$\text{Cutting speed} = \pi \times D \times N = 3.1416 \times 8 \times 50 \times 60 = 75398.4 \text{ in. per hr.}$$

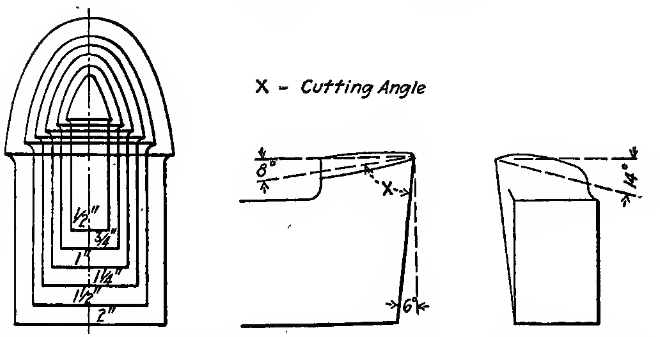
$$W = 75398.4 \times d \times t \times 0.26 = 75398.4 \times 1/32 \times 1/8 = 70.686 \text{ lbs.}$$

$$\text{h.p.} = YZW = 1 \times 0.035 \times 70.686 = 2.474 \text{ (Ans.).}$$

The **Average Cutting Force** at the edge of tool can be found by the following formula:

$$F = \frac{33000\,YZW}{S} \times 0.85 = \frac{28050\,YZW}{S}.$$

F = cutting force.
S = cutting speed in ft. per min.



Taylor Standard Cutting Contours

Back Rake & Clearance for Medium Steel & Iron

Side Slope for Medium Steel and Iron

STANDARD LATHE TOOL

Example 2: What is the pressure exerted upon the cutting tool in Example 3?

$$W = \frac{1{,}603{,}000\,YZW}{S} = \frac{28{,}050 \times 2.474}{\dfrac{75398.4}{12 \times 60}} = 664.3 \text{ lbs. (Ans.).}$$

The proper **Cutting Speed** of a lathe with modern high speed tools, can be found by using the following formula:

$$V = \frac{HS}{(\sqrt[3]{D + Y})(\sqrt[2]{F - Z})}.$$

V = cutting speed in feet per minute.
D = depth of cut, taking $1/64''$ as a unit.

F = feed, taking 1/64″ per revolution as a unit.
H = constant for hardness of material to be cut.

Hard cast iron or steel 0.6.
Medium cast iron or steel 1.0.
Soft cast iron or steel 2.0.

S = constant for size of tool:

232 for $\frac{3}{4}''$ sq. tool on cast iron.
215 for $\frac{1}{2}''$ sq. tool on cast iron.
325 for $\frac{3}{4}''$ sq. tool on steel.
288 for $\frac{1}{2}''$ sq. tool on steel.

Y = constant:

3 for $\frac{3}{4}''$ sq. tool on cast iron.
8 for $\frac{1}{2}''$ sq. tool on cast iron.
−2 for $\frac{3}{4}''$ sq. tool on steel.
0 for $\frac{1}{2}''$ sq. tool on steel.

Z = constant:

0 for $\frac{3}{4}''$ sq. tool on cast iron.
0.3 for $\frac{1}{2}''$ sq. tool on cast iron.
0.3 for $\frac{3}{4}''$ sq. tool on steel.
0.5 for $\frac{1}{2}''$ sq. tool on steel.

With high carbon tool steel the cutting speed is one half of the above amount.

Example: Find the cutting speed of a $\frac{3}{4}''$ square high speed steel tool in a lathe when the depth of cut is 3/16″ and the feed per rev. is 1/64 upon a piece of soft steel.

$$H = 2.0, \quad S = 325, \quad D = 12,$$
$$Y = -2, \quad Z = 0.3, \quad F = 1,$$

$$V = \frac{2 \times 325}{(\sqrt[3]{12} - 12)(\sqrt[2]{1} - 0.3)} = \frac{464}{2.15 \times 0.84}$$
$$= 360 \text{ ft. per min. (Ans.).}$$

The Approximate Horse Power Electric Motor Required to Drive Various Types of Machines

Drill presses

Sensitive—drills up to $\frac{1}{2}''$,	$\frac{1}{4}$ to $\frac{3}{4}$ h.p.
$12''$ to $20''$	1 "
$24''$ to $28''$	2 "
$30''$ to $32''$	3 "

Lathes

$6''$ to $10''$	1 h.p.
$12''$ to $14''$	1 to 2 "
$16''$ to $20''$	2 to 3 "
$22''$ to $27''$	3 to 5 "
$30''$ to $36''$	$7\frac{1}{2}$ to 10 "

Universal milling machines

No. 1	1 to 2 h.p.
" $1\frac{1}{2}$	2 to 3 "
" 2	3 to 5 "
" 3	5 to $7\frac{1}{2}$ "
" 4	$7\frac{1}{2}$ to 10 "

Shapers

$10''$ to $14''$ stroke	1 to 2 h.p.
$16''$ to $18''$ "	2 to 3 "
$20''$ to $24''$ "	3 to 5 "
$30''$ "	5 to $7\frac{1}{2}$ "

Planers

$22''$	3 h.p.
$24''$ to $27''$	3 to 5 "
$30''$	5 to $7\frac{1}{2}$ "
$36''$	10 to 15 "
$42''$	15 to 20 "

Gear cutters

$36'' \times 9''$	2 to 3 h.p.
$48'' \times 10''$	3 to 5 "
$30'' \times 12''$	5 to $7\frac{1}{2}$ "
$72'' \times 14''$	$7\frac{1}{2}$ to 10 "

Grinders

$8''$ to $10''$ wheel	5 h.p.
$12''$ to $14''$ wheel	$7\frac{1}{2}$ "
$16''$ to $20''$ "	10 "

EXERCISES

1. Find the horse power required to drive a lathe that cuts 100 lbs. of cast iron chips per hour.

2. What is the horse power required to drive a milling machine running at 100 r.p.m. working on a cast iron casting with $0.040''$ feed per rev., cut being $\frac{1}{4}'' \times 3''$?

3. What horse power will be required to run a lathe at 40 r.p.m. to turn a shaft to $2''$ in diameter, with a $\frac{1}{32}''$ feed, and $\frac{3}{16}''$ depth of cut?

4. What is the horse power required to drill a $1''$ hole in wrought iron, using a $0.010''$ feed, running at 55 r.p.m.

5. What is the pressure exerted upon the ram of a shaper taking a cut in tool steel $\frac{1}{8}''$ deep and $\frac{1}{32}''$ feed, cutting at a surface speed of 10 ft. per min.?

6. Find the proper cutting speed for a $\frac{3}{4}''$ sq. high speed steel tool in a lathe, with the following conditions:

material = C.I. of medium hardness
depth of cut = $\frac{3}{16}''$
feed per rev. = $\frac{1}{16}''$

7. Also find the cutting speed with a $\frac{3}{4}''$ sq. carbon steel tool:

material = hard cast iron
depth of cut = $\frac{1}{8}''$
feed per rev. = $\frac{1}{64}''$

8. What is the proper feed for a $\frac{1}{2}''$ sq. high speed steel tool, when the depth of cut is $\frac{1}{16}''$ on a piece of medium hard steel 8″ in diameter, running at 120 r.p.m.?

Force, Work, Energy and Momentum

Force is anything that tends to change the state of a body, whether at rest or in motion. A force means the pull, push rub, attraction or repulsion of one body upon another, and there is always a simultaneous, equal and opposite force called the reaction exerted by the second body upon the first.

Inertia is the property of a body by virtue of which it tends to continue in its present state of rest or motion until acted upon by some force.

The **Mass** of a body is the amount of matter it contains. It is equal to its weight divided by the attraction of gravity at that particular point.

$$M = \frac{W}{g}, \text{ where } \begin{array}{l} M = \text{mass.} \\ W = \text{weight.} \\ g = \text{attraction of gravity.} \end{array}$$

Momentum is a term used to designate the product of the mass of the body and its velocity at any instant. It represents the quantity of motion of a body.

$$\text{Momentum} = MV = \frac{WV}{g}, \text{ where } \begin{array}{l} W = \text{weight of body in lbs.} \\ M = \text{mass of body.} \\ V = \text{velocity in ft. per sec.} \end{array}$$

The rate of change of momentum is proportional to the force applied.

The rate of change of velocity or **Acceleration** of a body caused by a force is proportional to the force applied and inversely proportional to its mass.

$$a = \frac{M}{F} \text{ or } F = M \times a, \quad \begin{aligned} F &= \text{force in lbs.} \\ a &= \text{acceleration in ft. per sec. per sec.} \\ M &= \text{mass.} \end{aligned}$$

Example 1: If a man pushes a truck weighing 1½ ton with a force of 50 lbs. and the resistance of truck is 20 lbs. per ton weight, how long will it take to attain the velocity of 8 miles per hour?

$$\text{Accelerating force} = 50 - (20 \times 1\tfrac{1}{2}) = 20\text{lbs.}$$

$$a = \frac{F}{M} = \frac{20}{\dfrac{1\frac{1}{2} \times 2000}{32.16}} = 0.21 \text{ ft. per sec. per sec.}$$

$$8 \text{ miles per hour} = \frac{5280 \times 8}{60 \times 60} = 11.73 \text{ ft. per sec.}$$

$$\text{time} = \frac{\text{velocity}}{\text{acceleration}} = \frac{11.73}{0.21} = 55.9 \text{ sec. (Ans.).}$$

Energy is a capacity of doing work. It is of 2 kinds, **Potential** and **Kinetic.**

Potential energy is energy possessed by a body by virtue of its condition or position. A weight suspended above the ground or a body of water held by a dam possesses potential energy. Potential energy also exists as chemical energy in storage batteries, etc.

Kinetic energy is the energy possessed by a body by virtue of its motion, and is the work it is capable of performing against a retarding resistance before being brought to rest. A moving body, a flywheel, a current of air, or a falling body, all possess kinetic energy. The kinetic energy of a body is equal to the work done upon it to bring it from rest to its initial velocity.

Energy exists in several different forms, but the amount of energy in the universe is fixed and unchangeable. It may be transformed from one form to another, but none can be created or none destroyed.

Potential Energy is equal to the product of the force tending to produce motion and the distance through which the body is able to move.

$$E_P = W \times H.$$

E_P = potential energy.
W = weight of body.
H = height above ground.

Kinetic energy is obtained by multiplying one-half of its mass by the square of its velocity in ft. per sec.

$$E_K = \tfrac{1}{2}MV^2 = \frac{WV^2}{2g} = \frac{WV^2}{64.32}.$$

E_K = kinetic energy.
M = mass of body.
W = weight of body.
V = velocity of body in ft. per sec.

Example 2: What is the potential energy of a stone weighing 20 lbs. placed upon a roof 30 ft. high?

$$E_P = W \times H = 20 \times 30 = 600 \text{ ft. lbs. (Ans.).}$$

Example 3: If a ball weighing 3 lbs. is thrown vertically upward with a velocity of 100 ft. per sec., what is the kinetic energy possessed at the start?

$$E_K = \frac{WV^2}{2g} = \frac{3 \times 10000}{64.32} = 466.4 \text{ ft. lbs. (Ans.).}$$

Work is the overcoming of resistance through space. If no movement is produced no work is done, thus a jack screw supporting a load does no work unless the screw is turned.

Work is equal to the product of the force and the space

through which it acts. The **Unit** of **Work** is the ft. lb. or the work done by a force of one lb. acting through a distance of one ft.

$$W = FS.$$

F = force in lbs.
S = distance in feet.
W = work in ft. lbs.

Example 4: Find the work done by the charge, on a projectile weighing 100 lbs. which leaves the muzzle of a cannon with a velocity of 1000 ft. per second.

The work done by the projectile is equal to the kinetic energy possessed at the breech of the cannon.

$$\text{Therefore work done} = \frac{WV^2}{2g} = \frac{100 \times 1{,}000{,}000}{64.32}$$

$$= 1{,}554{,}710.8 \text{ ft. lbs. (Ans.).}$$

The energy of a falling body is equal to the weight multiplied by the height through which it falls.

The **Force of a Blow** cannot be expressed directly in pounds, but it can be expressed by the average force of blow.

The average force of blow is equal to the number of foot pounds divided by the amount of penetration, plus the weight of falling body.

$$\text{Average force of blow} = F = \frac{WS}{d} + W.$$

W = weight of falling body in lbs.
S = height in feet.
d = distance of penetration in ft.

Example 5: A single acting steam hammer weighing 500 lbs. falls through a distance of 4 ft. and compresses the work $\frac{1}{4}''$. What is the average force of blow?

$$W \times S = 500 \times 4 = 2000 \text{ ft. lbs.}$$

$$F = \frac{2000}{d} + W = \frac{2000}{\frac{1}{4} \times \frac{1}{12}} + 500 = 96{,}500 \text{ lbs. (Ans.).}$$

Power is the rate of doing work and is measured by the amount of the work divided by the time in which it is done. The unit of power is the horse power (h.p.) which is doing work at the rate of 550 ft. lbs. in one sec. or 33000 ft. lbs. in one minute.

$$P = \frac{FS}{t}.$$

F = force in lbs.
S = distance in ft.
t = time in seconds.

Example 6: A motor truck weighing 7 tons, attains a speed of 15 miles per hour from rest in 20 seconds, during which it travels 330 ft., the average resistance of the truck being 80 lbs. per ton. Find the average horse power used to drive the truck.

$$\text{acceleration} = \frac{\text{velocity}}{\text{time}} = \frac{15 \times 5280}{20 \times 60 \times 60}$$
$$= 1.1 \text{ ft. per sec. per sec.}$$

$$\text{accelerating force} = Ma = \frac{7 \times 2000}{32.16} \times 1.1 = 478.8 \text{ lbs.}$$

resistance = 80 × 7 = 560 lbs.
total force = 478.8 + 560 = 1038.8 lbs.

$$\text{h.p.} = \frac{F \times S}{t \times 550} = \frac{1038.8 \times 330}{20 \times 550} = 31.2 \text{ h.p. (Ans.).}$$

EXERCISES

1. A shell weighing 100 lbs. is fired vertically upward with a velocity of 1500 ft. per sec. What is its kinetic energy at the muzzle of the gun?

2. What is the potential energy of the above shell when it reaches its highest point?

3. A baseball weighing 9 oz. is dropped from the top of the Washington Monument, which is 550 ft. high. What is the energy possessed by the ball when it strikes the ground?

4. A railroad train weighing 100 tons is moving at the rate of 30 miles per hour. What is the momentum?

5. An elevator has an upward acceleration of 3.216 ft. per sec. per sec. What pressure will a man weighing 150 lbs. exert upon the floor of the elevator?

6. A waterfall is 65 ft. high. If 12 tons of water flow over the falls in 1 minute, what kw. generator will the falls run?

7. What horse power would a turbine develop if it received all the water flowing at a rate of 3 miles per hour over a dam 25 ft. high, if the water flowed 6″ deep over a dam which is 20 ft. long?

8. A motor truck weighing 7 tons is running at a uniform velocity of 15 miles per hr. What is the horse power required, if the traction resistance of the road is 110 lbs. per ton?

9. A train traveling at a speed of 30 miles per hour is brought to rest by a uniform resisting force within a distance of 3000 ft. What is the total resisting force in lbs. per ton?

10. What horse power will be required to run a train weighing 200 tons at the rate of 20 miles per hour, if the traction resistance is 10 lbs. per ton?

11. A locomotive has a total weight of 40 tons on the driving wheels and the coefficient of friction between the wheels and rails is 0.15. What is the maximum pull of the train?

12. A motor car weighing 3000 lbs. coasts down a slope of 1 in 30 at the rate of 15 miles per hour. What is the resistance of the load? What is the horse power required to ascend a slope at the same speed? (Friction being neglected.)

13. A stone weighing 50 lbs. falls through a distance of 20 ft. and sinks into the ground 15″ deep. What is the average force of blow?

14. A drop hammer weighing 500 lbs. falls through a distance of 4 ft. and compresses the work $\frac{1}{2}$″. What is the force of blow?

15. At what velocity must a body weighing 5 lbs. be moving in order to have stored in it 60 ft. lbs. of energy?

16. A motor truck weighing $1\frac{1}{2}$ tons is traveling at the rate of 20 miles per hour, pulling a 7 ton trailer. If the traction resistance of the road is 50 lbs. per ton, what is the necessary load on the rear axle, if the adhesion of the driving wheels is equal to $\frac{1}{2}$ of its load. Also what horse power motor is required in the truck?

Force Diagrams

The construction of **Force Diagrams** somewhat simplify the calculation of stresses on beams, etc.

According to one of the laws of motion, every action or force has an equal and opposite reaction; thus when a force or load is acting downward on a beam the supports have an equal upward reaction.

In a simple beam which is loaded in the center the reaction at each support is equal to one-half of the load. However, if the force or load is not central or uniformly distributed the reaction at the supports will vary.

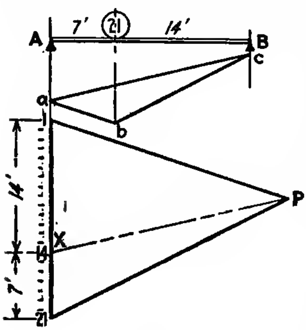


Fig. I

Example: In Fig. I a load of 21 pounds is resolved into two parallel components A and B, a distance of 7 and 14 feet, respectively, from the point of load. Find the magnitude of force at A and B in pounds.

First draw a vertical line 1–21 to represent 21 lbs. in any convenient unit of length, as shown, each unit representing one pound.

Next choose a convenient point P and connect points 1 and 21 with same.

Choose a convenient point a on line A–1 and draw line a–b parallel to 1–P intersecting line F–b.

Draw line b–c parallel with 21–P starting from point b. Join a and c with a straight line.

Then by drawing the line X–P parallel with a–c it will intersect or divide the line 1–21 in the same proportion as the load F is distributed at points of support A and B.

The distance from 1 to X measured in the same scale that 1–21 was laid out, *i.e.*, 14 units (14 lbs.) will be the magnitude of force at support B.

Example: Construct a force diagram of a bridge 40 feet long with a load of 1000 pounds located 10 feet from one end.

Solution: Fig. II.

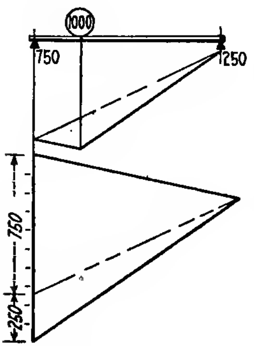


Fig. II

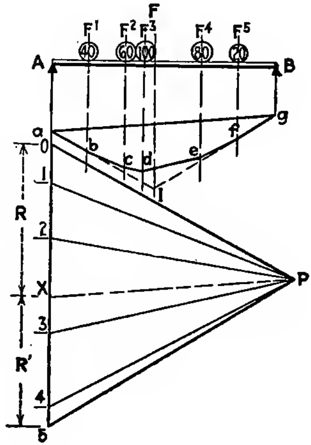


Fig. III

A force diagram for a simple beam which is not uniformly loaded is as follows:

Example: Construct a force diagram for a simple beam 12 feet long, loaded as shown in Fig. III.

Solution: First draw the beam to some convenient scale, say $\frac{1}{8}''$ per foot and locate the various loads F^1, F^2, F^3, F^4 and F^5 in their proper position with reference to each other.

Next draw a vertical line 0–5 to some convenient scale, making 0–1 represent 40 lbs., 1–2 equal 60 lbs., 2–3 equal 100 lbs., 3–4 equal 80 lbs. and 4–5 equal 20 lbs.

As in Fig. I choose a convenient point P and connect points 0, 1, 2, 3, 4 and 5 to P.

Then choose a convenient point a on line A–0 and construct the polygon a–b–c–d–e–f and g, these various lines being parallel with lines 0–P, 1–P, 2–P, 3–P, 4–P and 5–P, respectively.

If the lines a–b and e–f are continued until they intersect at I, this will give the position of the line of the resultant force F; *i.e.*, a point where the total load is concentrated or a point where the beam would balance.

Connecting points a and g with a straight line and constructing line X–P parallel to same will divide line 0–5 in proportion to the distribution of the loads at point A and B.

Thus 0–X or R when measured off will equal 160 lbs. or the reaction at support A and X–5 or R' will equal 140 lbs. or the reaction at support B.

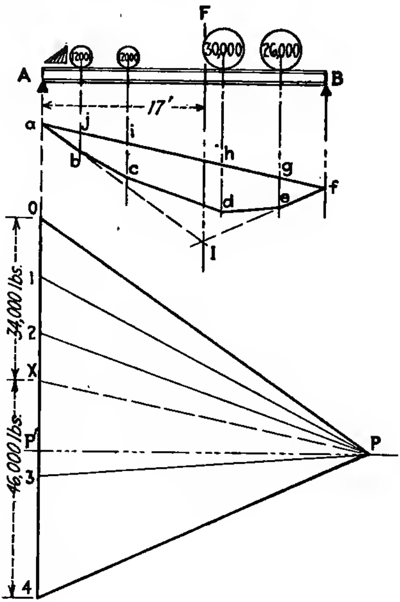


FIG. IV

The sum of all the reactions is always equal to the sum of all the loads. As shown in Fig. III the sum of the reactions, *i.e.*, 160 and 140 lbs., is equal to the total load or 40 + 60 + 100 + 80 + 20 lbs.

Example: Construct a force diagram of a locomotive which has its weight distributed as shown in Fig. IV; also the resultant reactions at points *A* and *B* and the distance the resultant force will be from *A*.

Solution: Fig. IV.

Example: Construct a force diagram of a beam loaded as shown in Fig. V.

Solution: First draw the beam to some convenient scale and locate the various loads and points of support in their proper position with respect to one another.

Next draw the vertical force line 0–3 to some convenient scale making 0–1 equivalent to 1000 lbs., 1–3 equals 3000 lbs. and 2–3 equals 1000 lbs.

Then connect points 0, 1, 2 and 3 to *P* which, as previously stated, can be located at any convenient position.

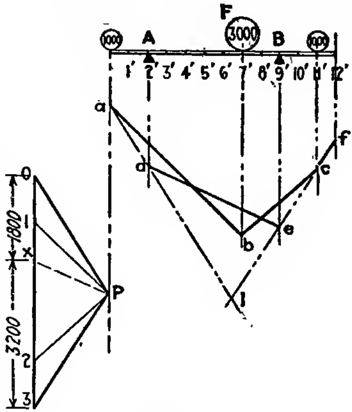


FIG. V

Construct the force diagram line *a–b* and *c* parallel to lines 0–*P* and 3–*P* respectively.

Draw *a–d* and *I* parallel with line 0–*P* beginning at *a*, also draw line *I–e–c* and *f* parallel with line 3–*P* intersecting the line *a–b* and *c* at *c*.

A vertical line projected upward from their intersections, *i.e.*, from *I* to *F* will give the line of the resultant force.

By drawing a straight line between points *d* and *e* (which are the points where the lines *a–d–I* and *I–e–c–f* intersect the vertical lines of support *A–d* and *B–e* respectively), and drawing line *X–P* parallel to same, will intersect or divide the line 0–3 at *X* proportional to the reaction at supports *A* and *B*.

16

Thus the distance o–X is equivalent to 1800 lbs. or the reaction at A and X–3 is equal to 3200 lbs. or the reaction at B.

Shear Diagram

A **Vertical Shear Diagram** may be readily constructed from a force diagram.

For instance at the left hand end of Fig. VII there is a 40 lb. force tending to force the beam downward, while at point A there is a reaction or upward force of 40 lbs.

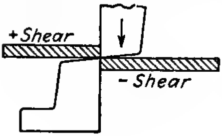


FIG. VI

These two forces acting in opposite directions tend to shear the beam.

In a simple beam the shear at opposite ends is equivalent to the reaction at these points and the greatest positive shear is at the left hand end and the greatest negative shear is at the right hand end.

To construct a vertical shear diagram of a beam, as shown in Fig. VII, the force diagram should be constructed first as previously explained. (See Fig. III.)

From X draw the horizontal line X–X' called the **Shear Axis** as shown in Fig. VII.

The shear diagram above the shear line is positive and that below the shear line is negative, *i.e.*, tending to produce shear by rotating in the clock-wise direction (such as it would to the right of the point of support in Fig. VII), is called positive shear; and tending to produce shear by rotating counterclock-wise (such as would be produced to the left of the support) is called negative shear.

The vertical shear at any point is equal to the reaction at the left hand support, less all the magnitude of forces or loads on the beam to the left of the force or load in consideration.

The amount of shear between A and $2'$ is constant and is equal to the reaction at point A or 40 lbs., and a line drawn horizontally from 0 out as far as the 40 lb. weight extends (represented by the line 0–a) will be the shear line for the 40 lb. weight.

At any point between $2'$ and $4'$ the shear will be partially counterbalanced by the 40 lb. weight and thus the total shear between points $2'$ and $4'$ will be equal to 160 lbs. − 40

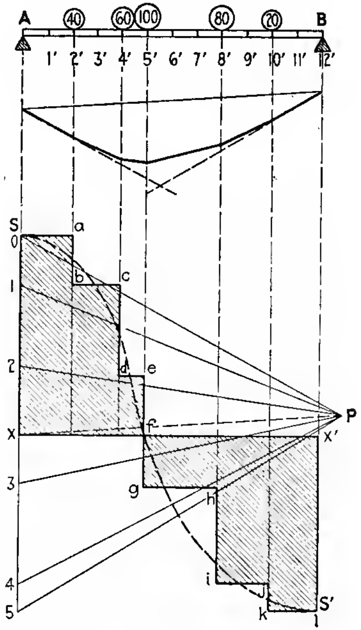


FIG. VII

lbs. or 120 lbs.; and a line drawn opposite 1 and extending between the 40 and 60 lb. weight, as shown by the line b–c, will be the shear line for the 60 lb. weight.

At any point between 4′ and 5′ the shear will be equal to 200 lb. − (40 + 60 or 100 lbs.) equals 100 lbs. and a line *d–e* drawn opposite 2 and underneath 4′ and 5′ will be the shear line for the 100 lb. weight, etc.

Then a line connecting the remainder of the points *f–g–h i–j–k* and *l* will be the shear line and the shaded portion will be the shear diagram.

The portion above the line *X–X′* will be positive and that below the line will be negative.

In reality the shear line will not be a broken line as shown but it will be a curved line, taking the mean average as shown by the dotted line *S–S′*.

Bending Moment

The bending moments of a beam is the measure of the tendency to produce rotation about a given point, caused by an external force. It is equal to the algebraic sum of the moments of all the external forces acting on one side of the section of a beam.

Positive (+) bending moments are those which tend to produce rotation clockwise on the section of the beam to the left of a given point and those that tend to produce motion counter-clockwise are **Negative** (−).

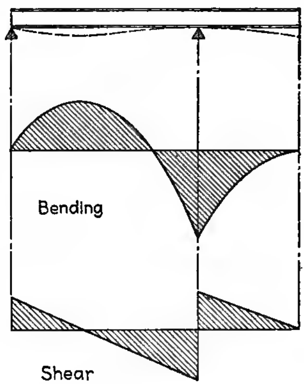


FIG. VIII

It is therefore evident that positive bending moments will produce convexity upward and negative bending moments convexity downward, see Fig. VIII.

If the force or load is measured in pounds and the distance

in feet the moment is expressed in foot pounds, but if the distance is measured in inches the moment is expressed in inch pounds. The latter is most frequently used.

Bending moments can be quickly and conveniently determined graphically by means of funicular polygons.

In Fig. IV the polygon *a–b–c–d–e–f–g–h–i* and *j* is called the **Bending Moment Diagram,** and the bending moment at any point of the beam can be found by multiplying the depth of the bending moment diagram at the point in consideration by the distance $P'-P$, which is the distance from the line 0–4 to P measured at right angles.

Thus the distance between points *h–d* measured to the same scale that 0–4 was originally laid out to, times the distance $P'P$ in inches will be the bending moments in inch pounds at that particular point.

If *h–d* is equal 10,000 lbs. and $P'-P$ is equal to 3.5″ drawn to 1/120 size, then the bending moments in inch pounds will be equal to 10,000 × 3.5 × 120 or 4,200,000 inch pounds.

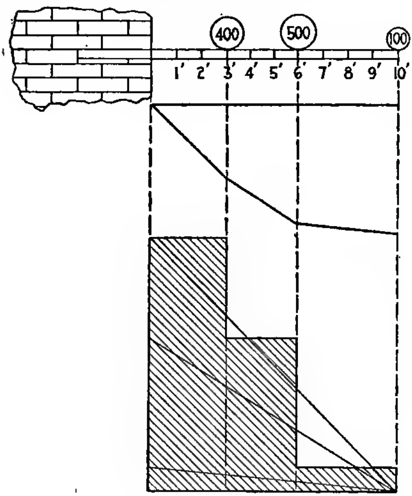


FIG. IX

At the other points it will be in proportion to the depth of the bending moment diagram.

Example: Construct a force, vertical shear and bending

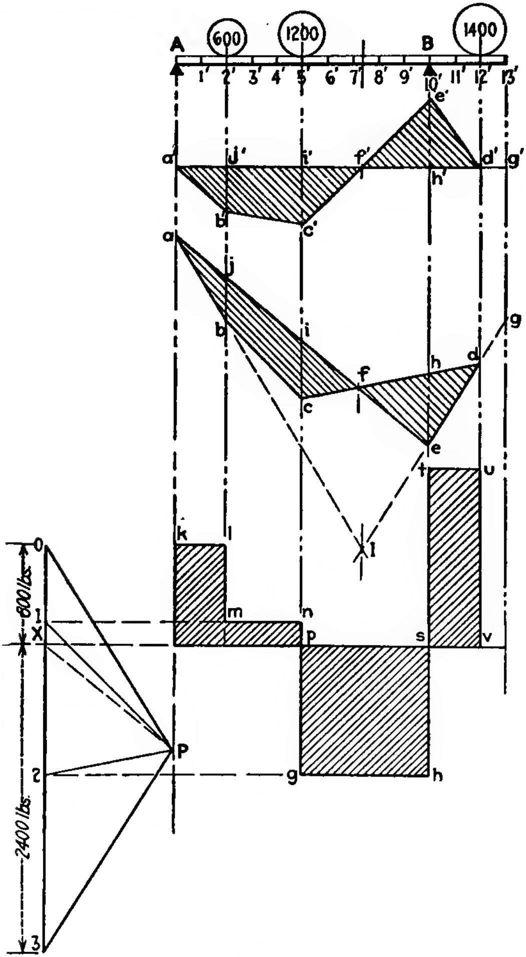


Fig. X

moment diagram of a cantilever beam projecting 10 ft. out from a wall and loaded with a 400 lb. weight 3 ft. from the wall, a 500 lb. weight 6 ft. from the wall and a 100 lb. weight at the extreme end.

Solution: Fig. IX.

Example: Construct a force, vertical shear and a bending moment diagram of a beam 13 feet long, supported at the extreme left and also at a point 3 feet from the right hand end, with a load of 600 lbs. 2 feet from the left end, a load of 1200 lbs. 5 feet from the left end and a load of 1400 lbs. 12 ft from the left end.

Solution: Fig. X.

The distances 0–1, 1–2 and 2–3 are laid out representing 600, 1200 and 1400 lbs. respectively to any convenient scale.

Points 0–1–2 and 3 are connected to point P which has been arbitrarily selected at a point directly underneath $A–a$ and an equal distance between points 0–3.

The line 0–P is transferred to the polygon and represents line $a–b–1$, 1–P represents $b–c$, 2–P represents $c–d$, and 3–P represents line $I–e–d$ and g, which intersects line $c–d$ at d, or a continuation of line $12'–d'$.

The points a and e, (*i.e.*, where the lines $a–b–I$ and $I–e–d–g$ intersect the vertical support lines $A–a'–a$ and $B–e'–e$) are then connected by a straight line $a–e$.

This line is transferred to $X–P$ and divides the line 0–3 in proportion to the distribution of the weights on supports A and B, which is approximately 800 lbs. at A and 2400 lbs. at B.

It should be noted that line $a–e$ intersects line $c–d$ at f. This denotes that the moment to the left of f is positive and that to the right of f is negative.

To make the bending moment diagram clearer it is frequently transferred to a parallel base line $a'–g'$ and the positive bending moment is placed below the line and the negative

bending moment is placed above the line as shown in the upper bending moment diagram.

In this case it is found that the maximum negative bending moment is greater than the maximum positive bending moment, and the beam should be so designed that it will withstand the maximum moment whether positive or negative.

To construct the shear diagram the shear line x–p–s–v is constructed and, as previously stated, the positive shear, or that to the right, is placed above the shear line.

In this case there are two positive shear diagrams, one to the right of each support as shown above the shear line while there is only one negative shear which is shown below the line.

Note that the reversal of the shear takes place directly underneath the maximum bending moment in all cases.

In Fig. X the two reversals of shear at p and s take place directly underneath the maximum positive and negative bending moments i'–c' and $e'h'$.

Also in Fig. VII point f or the reversal of the shear takes place beneath the maximum bending moment.

Pendulum

A **Simple Pendulum** is an imaginary one consisting of a heavy point suspended by a weightless string.

A Compound Pendulum is a material body, suspended from a fixed axis, about which it oscillates or swings by the force of gravity.

The center of oscillation of a compound pendulum is the point at which, if all the matter in the pendulum were concentrated there, it would make a simple pendulum which would vibrate or oscillate in the same period of time.

The angle included between the extreme positions of a

line drawn from the point of suspension to the center of oscillation is called the **Angle of Oscillation.**

The **Time of Vibration** of a pendulum depends on its length and the acceleration of gravity at the given latitude and elevation above the sea level. The time of vibration of a pendulum varies directly as the square root of its length and inversely as the square root of the acceleration of gravity at the given point. The time of vibration of a pendulum may be varied by adding a weight above the point of suspension which counteracts the lower weight and lengthens the time of vibration.

A pendulum of a given length always vibrates in the same time period at a given locality, provided the angle of oscillation does not exceed 5 deg. This property of a pendulum is called its **Isochronism.**

If a weight suspended by a cord revolves at a uniform speed along the circumference of a circle in a horizontal plane, this weight forms a conical pendulum, and it is held in equilibrium by three forces, the tension in the cord, the centrifugal force, and the force of gravity.

To find the time of vibration of a compound pendulum reduce it to an equivalent simple pendulum and find its time of vibration.

At New York City, a pendulum which is 39.1017″ long (3.2585 ft.) will vibrate seconds.

Abbreviations Used in Formulas

$W_1 W_2$ = Weight of balls in lbs.
W_3 = Weight of bar in lbs.
d = Distance of center of gravity from point of suspension.
L = Length of simple pendulum in inches.
m = Radius of gyration.
t = Time in sec. for "n" oscillations.

t_0 = Time in sec. for one revolution.
g = Value of acceleration of gravity.
v = Velocity in ft. per sec. of center of gravity of weight.
n = No. of single oscillation in "t" seconds.
n_0 = No. of rev. per min.
G = Center of gravity of whole weight.
P = Point of suspension.
O = Center of oscillation.
C = Center of gyration.
L_0 = Length of arm of conical pendulum in feet.
r = Radius of ball circle in ft. = $L_0 \sin \alpha$.
h = Distance of the ball circle below the point of support in ft.

Simple Pendulum

$$t = \frac{n\sqrt{L}}{6.25}. \qquad n = \frac{6.25t}{\sqrt{L}}.$$

$$L = \frac{m^2}{d}.$$

$$= \frac{12gt^2}{\pi^2 n^2} = \frac{39.1t^2}{n^2}.$$

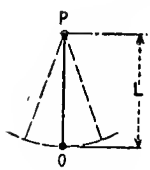


FIG. I

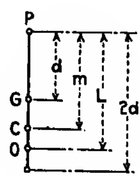


FIG. II

Compound Pendulum

$$d = \frac{d_1W_1 + d_2W_2 + d_3W_3}{W_1 + W_2 + W_3}.$$

$$m^2 = \frac{d_1^2 W_1 + d_2^2 W_2 + d_3^2 W_3}{W_1 + W_2 + W_3}.$$

$$L = \frac{m^2}{d} = \frac{d_1^2 W_1 + d_2^2 W_2 + d_3^2 W_3}{d_1 W_1 + d_2 W_2 + d_3 W_3}.$$

$$d = \frac{d_2 W_2 + d_3 W_3 - d_1 W_1}{W_1 + W_2 + W_3}.$$

$$m^2 = \frac{d_1^2 W_1 + d_2^2 W_2 + d_3^2 W_3}{W_1 + W_2 + W_3}.$$

$$L = \frac{m^2}{d} = \frac{d_1^2 W_1 + d_2^2 W_2 + d_3^2 W_3}{d_2 W_2 + d_3 W_3 - d_1 W_1}.$$

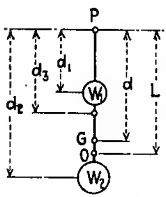


FIG. III

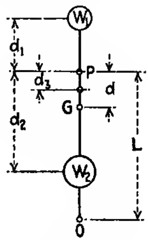


FIG. IV

Conical Pendulum

$$t_0 = \frac{2\pi r}{v} = 6.283\sqrt{\frac{g}{h}} = \frac{60}{n_0}.$$

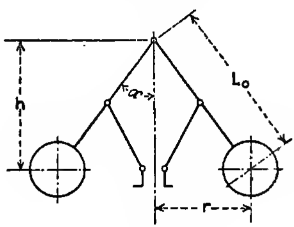


FIG. V

$$n_0 = \frac{60}{2\pi}\sqrt{\frac{g}{h}} = \frac{54.2}{\sqrt{h}} = \frac{54.2}{\sqrt{L_0 \cos \alpha}}.$$

$$h = \frac{g t_0^2}{4\pi^2} = 0.8146 t_0^2 = \frac{2933}{n_0^2} = L_0 \cos \alpha.$$

EXERCISES

1. What is the length of a simple pendulum to beat seconds where the attraction of gravity is 32.16 ft. per sec. per sec.?

2. What is the attraction of gravity at a point where a simple pendulum 39.1393″ long, beats seconds?

3. What is the time of vibration of a simple pendulum 3 ft. long, when $g = 32.16$?

4. If a simple pendulum is 48″ long and $g = 32.16$, how many beats would it make in 20 seconds?

5. If a compound pendulum is composed of a brass disk ½″ thick and 6″ in diameter, the rod is ¼″ in diameter and 66″ long, what is the time of one beat if $g = 32.16$?

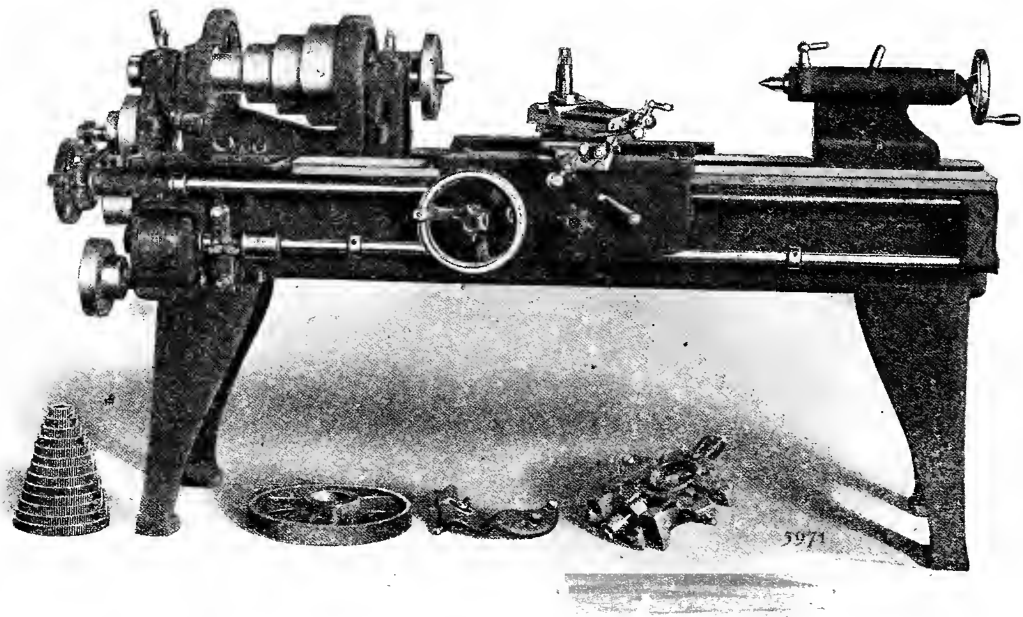
6. A compound pendulum consists of 2 cast iron balls 3″ and 6″ in diameter and hangs on a ½″ steel rod 46.8″ long. The small weight is 24.3″ from the point of support. What is the time of one beat if $g = 32.16$″?

7. What is the difference in the value of "g" between two points A and B, if a pendulum to beat seconds is 39.0152″ long at A and is 39.1393″ long at B?

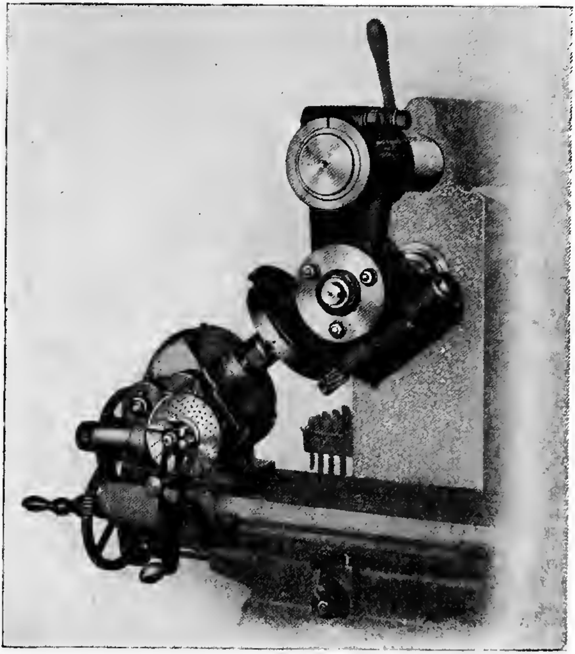
8. A round steel bar ¾″ in diameter and 5′ long is pivoted at a point 2/5 of its length from one end. On the short end is a cast iron ball 4″ in diameter and on the long end is a cast iron ball 7″ in diameter. If the rod just extends through the balls and $g = 32.16$, what is the time of the pendulum?

9. In a conical pendulum the angle $\alpha = 45$ deg. If the centers of the balls are 6″ from the pivot point, how many revolutions per minute is the pendulum making?

10. An engine governor has arms which are 7″ long and is turning at the rate of 80 revolutions per min.; find the time of one revolution; the radius of the ball circle; the distance of the ball circle below the point of support; and the angle at which the arms are standing.



Engine Lathe



CAM CUTTING

Cam Design

A cam is a mechanical device for converting rotary motion into reciprocating motion. Cams are made in several forms but usually consist of an irregular shaped disk or of a groove cut in a flat or curved surface.

Cams may be classified in two general classes: according to their shape, and also as to the motion they produce. As regards shape, we have heart cams, disk cams, face or plate cams, and barrel cams. As regards motion, we have uniform acceleration cams, harmonic or crank cams and intermittent cams.

A heart cam is the simplest form of cam, and converts rotary motion into uniform reciprocating motion. Disk cams may have any shape and generally act directly on the follower, depending on gravity or a spring to effect the return of the follower. Face or plate cams have an irregular groove in which the follower moves, cut on the face or plate. They may produce any form of motion and have a positive return. Barrel cams are cams having a slot cut on the outside of a cylindrical surface and may produce any motion.

A **Uniform Velocity Cam** is one in which the follower is made to pass over equal spaces in equal lengths of time, or is impelled from rest to maximum velocity instantly and then brought to rest from maximum velocity instantly.

A **Uniform Acceleration Cam** or gravity cam is one in which the follower is brought from rest to a maximum velocity with a uniform acceleration and then brought to rest again with a uniform retardation. Since the movement of the follower is similar to that of a falling body, this is also called a gravity curve cam. A **Harmonic Cam** is one in which the follower is brought gradually from rest to maximum velocity and then gradually brought to rest, but the acceleration is not uniform. This is also called a crank

cam since the action of the follower is very similar to that of a crank.

An **Intermittent Cam** may have any irregular movement.

Uniform velocity cams can only be used at low speeds because the abrupt changes from rest to movement and vice versa cause too great a shock to the machine at high speeds.

Harmonic and gravity cams are both used for high speed work for these cams produce a very smooth working movement. The harmonic curve, while easy to design does not give as smooth action as the gravity curve, but either gives good results.

Intermittent cams are used wherever the service demands such a movement, and generally combine periods of rest and periods of movement without regard to any set rule.

There are three distinct phases to the movement of a follower, advance, retreat and dwell or rest, and any combination of these three which returns the follower to its starting point, constitutes a cycle.

If we consider a movement of the follower in any one direction as an advance, then any movement in the opposite direction is a retreat.

When a cam, though rotating, produces no movement of the follower, the follower is said to dwell or rest.

To lay out a heart shaped, uniform velocity cam (Fig. I). Follower R is to be given a reciprocating motion equal to distance 1–I: let X be the center of the cam. With X as center, draw semi-circle A–S–1, and extend diameter A–X–1 to I, making 1–I equal the required throw; divide 1–I into any number of equal parts as 1, 2, 3, etc. and divide the semi-circle by as many radii equally spaced. With X as center and radius X–2, draw an arc intercepting X–B at B, with same center and radius X–3, draw an arc intercepting X–C at C. Continue this process through points 4, 5, 6,

etc. obtaining points D, E, F, etc. The latter are points on the required curve. The other half of cam is laid out in a like manner.

Since a pointed follower (Fig. I) would cause excessive friction, a roller is sometimes used to reduce the friction. The curve of the came must be modified slightly, as the curve in Fig. I would not give the proper travel to the roller

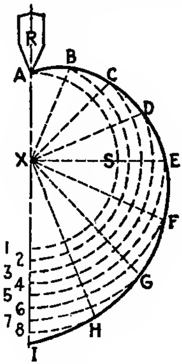


FIG. I

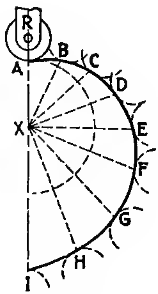


FIG. II

follower in Fig. II. It is the path of the center of this roller which must be considered, as the position of this center regulates the throw. The position of this center may be determined by adding to each of the distances $X-B$, $X-C$, etc. in Fig. I, the radius of the roller R in Fig. II, thus obtaining the points B, C, etc. in Fig. II. Using these points as centers with radius equal to that of the roller, describe arcs. A curve drawn tangent to these arcs is the desired curve.

To design a uniform acceleration cam (Fig. III) having a throw equal to 1–G, and an acceleration of two units per unit of time, draw semi-circle A–S–1, and divide the circumference into 6 equal parts. Project diameter A–X–1

to *G* making 1–*G* equal to the desired throw. Divide 1–*G* into two equal parts as 1–4 and 4–*G*, and subdivide these parts into three divisions whose lengths are to each other as 1, 3, 5. With *X* as a center, draw arcs from 1, 2, 3, etc. to intercept the radii in points *A*, *B*, *C*, etc. which are points in the cam curve for a pointed follower. Since the follower carries a roller, the radius *A–R* of the roller must be added to lengths of *X–B*, *X–C*, *X–D*, etc. and arcs drawn from these points *K*, *L*, *M*, etc. using *AR* as radius. A curve, tangent to these arcs, is the desired cam curve. If we divide the half-circle into 8 equal parts, then we must divide 1–*G* into 8 parts also, in the proportion of 1, 3, 5, 7, 7, 5, 3, 1. With this cam, a follower starts at *R*, with velocity at zero, reaches its maximum velocity at *M*, and at *P*, where it reverses, its velocity is again zero, making a quiet easy working cam suitable for high speeds.

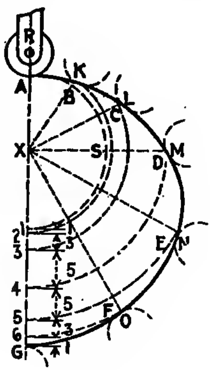


Fig. III

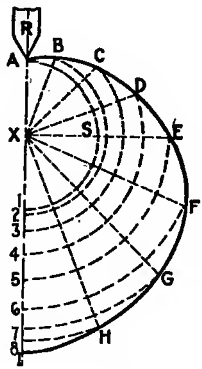


Fig. IV

To design a harmonic cam (Fig. IV), follower *R* is to be given a harmonic movement which means that *R* will be brought from rest to maximum velocity with a gradual acceleration and then brought to rest at the reversing point

with a gradual retardation. This cam differs from the gravity cam, in that the acceleration and retardation are not uniform but variable.

Draw semi-circle *A–S–*1 and extend diameter to *I*, making 1–*I* equal to the desired throw. Divide 1–*I* into any number of parts, as eight, whose lengths shall gradually increase from 1 to 5 and gradually decrease from 5 to *I*. Divide the semi-circle by as many radii equally spaced. Draw the arcs 2–*B*, 3–*C*, 4–*D*, etc., intercepting the radii at points *B*, *C*, *D*, etc. A curve passing through these points will be the desired curve. For a roller follower, add radius of roller to lengths *X–B*, *X–C*, *X–D*, etc. and proceed as in Fig. II. This cam can be used for high speeds, although it is not as easy working as a gravity curve cam.

Effect of changing location of cam roller

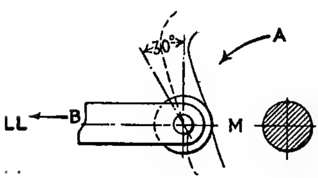


FIG. V

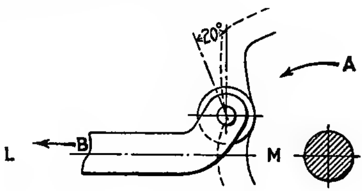


FIG. VI

When the line of motion of a follower passes through the center of the cam, and the angle of the cam causes it to work hard, the curve may be modified and the same movement of the follower obtained by placing the follower so that its line of movement is parallel to its former position, but not passing through the center of the cam. As example, in Fig. V: Here the cam, rotating in direction of arrow (*A*) moves the follower in direction of arrow (*B*). The angle of the cam with the follower at beginning of stroke is 30 deg. as determined by a tangent to the curve. Should the cam work hard, this could be remedied by increasing the diameter

of the cam which would reduce the angle of the cam. Sometimes this cannot be done owing to the design of the machine, but the same result can be accomplished by changing the location of the roller. Fig. VI has the same condition as Fig. V, but the cam roller has been moved above the line of center of the cam. The line of motion now passes along line $L-M$ and angle of cam is 20 deg., making an easier working cam. The roller must always be offset in the direction opposite to rotation of the cam and the angle of the cam decreases as the offset increases. However, if the follower be offset too much, the thrust at right angles to line of motion will increase the friction until the good effects of the offset are overcome.

EXERCISES

NOTE.—(Draw all cams on a 4″ diameter base circle.)

1. Lay out a heart-shaped, uniform velocity cam for a pointed follower, that will give a reciprocating motion of 2″ to the follower.

2. Lay out a similar cam to give the same movement to a follower with a roller $1\frac{1}{4}$″ in diameter.

3. Lay out a harmonic cam to give a reciprocating movement of $2\frac{1}{2}$″ to a follower with a roller $1\frac{1}{2}$″ in diameter.

4. Draw a gravity curve cam to give a reciprocating movement of $1\frac{1}{2}$″ to a roller follower: radius of roller $\frac{1}{2}$″ and an acceleration of 3.

5. Lay out a uniform velocity cam which shall advance $\frac{3}{4}$″ during a 30 deg. revolution of the cam; dwells for 60 deg., retreats $\frac{3}{4}$″ during next 30 deg. and dwells until the end of the cycle. Show (by sketch) that this cam will work easier if the roller is offset $\frac{3}{4}$″.

REVIEW EXERCISES

1. At what speed in r.p.m. must a 6″ grinding wheel run to attain a surface speed of 6000 ft. per minute?

2. At what surface speed in feet per minute is a 20″ grinding wheel running if the spindle runs at 1150 r.p.m.?

3. How many graduations must I have on my dial to represent 0.001″ movement of a tool on a cross slide of a lathe carriage, if an 8 pitch screw is used?

4. Find the number of graduations on a dial to read thousandths, on an elevating screw of a milling machine, with a reduction of 3 to 4 in bevel gearing, if a 6 P screw is used.

5. If the arm ratio of an indicator is $\frac{1}{2}$ to 11, what error does a $\frac{1}{64}''$ movement on the long arm represent?

6. What pressure is required to punch a hole 2″ in diameter in a soft steel plate $\frac{3}{8}''$ thick, if the shearing resistance of steel is 60,000 lbs.?

7. What weight will a $\frac{3}{8}''$ round steel rod support, under tension, if the steel has a tensile strength of 100,000 lbs. per square inch, providing a factor of safety of 8 is used?

8. A longitudinal steel boiler stay 20 ft. long and 1″ in diameter supports a flat area of 10″ square, having on it a pressure of 120 lbs. per sq. inch. Find the greatest stress in the stay due to its own weight and the steam pressure.

9. What must be the diameter of a steel car axle to resist the shearing stress of a load of 80,000 lbs., using factor of safety 15.

10. How many $\frac{3}{4}''$ studs must be used to hold a 24″ cylinder head of a steam engine, if the maximum steam pressure is 125 lbs. per sq. in., allowing a factor of safety of 8. The material from which the studs are made has a tensile strength of 60,000 lbs. per sq. inch. Use the root diameter of the studs as the effective area.

APPENDIX

TABLE I

Decimal Equivalents, Squares and Square Roots of Fractions

Fraction			Decimal Equivalent	Square	Sq. Root
		1/64	—0.015625	0.000244	0.1250
	1/32		—0.03125	0.000977	0.1768
		3/64	—0.046875	0.0022	0.2165
1/16			—0.0625	0.0039	0.2500
		5/64	—0.078125	0.0061	0.2795
	3/32		—0.09375	0.0088	0.3062
		7/64	—0.109375	0.0119	0.3307
1/8			—0.125	0.0156	0.3535
		9/64	—0.140625	0.0198	0.3750
	5/32		—0.15625	0.0244	0.3953
		11/64	—0.171875	0.0295	0.4161
3/16			—0.1875	0.0352	0.4330
		13/64	—0.203125	0.0413	0.4507
	7/32		—0.21375	0.0479	0.4677
		15/64	—0.234375	0.0549	0.4841
1/4			—0.250	0.0625	0.5000
		17/64	—0.265625	0.0706	0.5154
	9/32		—0.28125	0.0791	0.5303
		19/64	—0.296875	0.0881	0.5449
5/16			—0.3125	0.0977	0.5590
		21/64	—0.328125	0.1077	0.5728
	11/32		—0.34375	0.1182	0.5863
		23/64	—0.359375	0.1291	0.5995
3/8			—0.375	0.1406	0.6124
		25/64	—0.390625	0.1526	0.6250
	13/32		—0.40625	0.1650	0.6374
		27/64	—0.421875	0.1780	0.6495
7/16			—0.4375	0.1914	0.6614
		29/64	—0.453125	0.2053	0.6732
	15/32		—0.46875	0.2197	0.6847
		31/64	—0.484375	0.2346	0.6960
1/2			—0.5	0.2500	0.7071
		33/64	—0.515625	0.2659	0.7181
	17/32		—0.53125	0.2822	0.7289
		35/64	—0.546875	0.2991	0.7395
9/16			—0.5625	0.3164	0.7500
		37/64	—0.578125	0.3342	0.7603
	19/32		—0.59375	0.3525	0.7706
		39/64	—0.609375	0.3713	0.7806
5/8			—0.625	0.3906	0.7906
		41/64	—0.640625	0.4104	0.8004
	21/32		—0.65625	0.4307	0.8101
		43/64	—0.671875	0.4514	0.8197
11/16			—0.6875	0.4727	0.8292
		45/64	—0.703125	0.4944	0.8385
	23/32		—0.71875	0.5166	0.8478
		47/64	—0.734375	0.5393	0.8569
3/4			—0.750	0.5625	0.8660
		49/64	—0.765625	0.5862	0.8750
	25/32		—0.78125	0.6104	0.8839
		51/64	—0.796875	0.6350	0.8927
13/16			—0.8125	0.6602	0.9014
		53/64	—0.828125	0.6858	0.9100
	27/32		—0.84375	0.7119	0.9186
		55/64	—0.859375	0.7385	0.9270
7/8			—0.875	0.7656	0.9354
		57/64	—0.890625	0.7932	0.9437
	29/32		—0.90625	0.8213	0.9520
		59/64	—0.921875	0.8499	0.9601
15/16			—0.9375	0.8789	0.9682
		61/64	—0.953125	0.9084	0.9763
	31/32		—0.96875	0.9385	0.9843
		63/64	—0.984375	0.9690	0.9922
1			—1.000	1.0000	1.0000

TABLE II

Natural Trigonometrical Functions

D	*M*	Sines	Cosines	Tang.	Cotang.	*D*	*M*
0	0	.00000	1.0000	.00000	Infinite	90	0
	15	.00436	.99999	.00436	229.182		45
	30	.00873	.99996	.00873	114.589		30
	45	.01309	.99991	.01309	76.3900		15
1	0	.01745	.99985	.01745	57.2900	89	0
	15	.02181	.99976	.02182	45.8294		45
	30	.02618	.99966	.02618	38.1885		30
	45	.03054	.99953	.03055	32.7303		15
2	0	.03490	.99939	.03492	28.6363	88	0
	15	.03926	.99923	.03929	25.4517		45
	30	.04362	.99905	.04366	22.9038		30
	45	.04798	.99885	.04803	20.8188		15
3	0	.05234	.99863	.05241	19.0811	87	0
	15	.05669	.99839	.05678	17.6106		45
	30	.06105	.99813	.06116	16.3499		30
	45	.06540	.99786	.06554	15.2571		15
4	0	.06976	.99756	.06993	14.3007	86	0
	15	.07411	.99725	.07431	13.4566		45
	30	.07846	.99692	.07870	12.7062		30
	45	.08281	.99656	.08309	12.0346		15
5	0	.08716	.99619	.08749	11.4301	85	0
	15	.09150	.99580	.09189	10.8829		45
	30	.09585	.99540	.09629	10.3854		30
	45	.10019	.99497	.10069	9.93101		15
6	0	.10453	.99452	.10510	9.51436	84	0
	15	.10887	.99406	.10952	9.13093		45
	30	.11320	.99357	.11393	8.77689		30
	45	.11754	.99307	.11836	8.44896		15
7	0	.12187	.99255	.12278	8.14435	83	0
	15	.12620	.99200	.12722	7.86064		45
	30	.13053	.99144	.13165	7.59575		30
	45	.13485	.99086	.13609	7.34786		15
8	0	.13917	.99027	.14054	7.11537	82	0
	15	.14349	.98965	.14499	6.89688		45
	30	.14781	.98902	.14945	6.69116		30
	45	.15212	.98836	.15391	6.49710		15
9	0	.15643	.98769	.15838	6.31375	81	0
	15	.16074	.98700	.16286	6.14023		45
	30	.16505	.98629	.17634	5.97576		30
	45	.16935	.98556	.17183	5.81966		15
10	0	.17365	.98481	.17633	5.67128	80	0
	15	.17794	.98404	.18083	5.53007		45
	30	.18224	.98325	.18534	5.39552		30
	45	.18652	.98245	.18986	5.26715		15
11	0	.19081	.98163	.19438	5.14455	79	0
	15	.19509	.98079	.19891	5.02734		45
	30	.19937	.97992	.20345	4.91516		30
	45	.20364	.97905	.20800	4.80769		15
12	0	.20791	.97815	.21256	4.70463	78	0
	15	.21218	.97723	.21712	4.60572		45
	30	.21644	.97630	.22169	4.51071		30
	45	.22070	.97534	.22628	4.41936		15
13	0	.22495	.97437	.23087	4.33148	77	0
	15	.22920	.97338	.23547	4.24685		45
	30	.23345	.97237	.24008	4.16530		30
	45	.23769	.97134	.24470	4.08666		15
14	0	.24192	.97030	.24933	4.01078	76	0
	15	.24615	.96923	.25397	3.93751		45
	30	.25038	.96815	.25862	3.86671		30
	45	.25460	.96705	.26328	3.79827		15
15	0	.25882	.96593	.26795	3.73205	75	0

TABLE II—(*Continued*)

D	M	Sines	Cosines	Tang.	Cotang.	D	M
15	0	.25882	.96593	.26795	3.73205	75	0
	15	.26303	.96479	.27263	3.66796		45
	30	.26724	.96363	.27732	3.60588		30
	45	.27144	.96246	.28203	3.54573		15
16	0	.27564	.96126	.28674	3.48741	74	0
	15	.27983	.96005	.29147	3.43084		45
	30	.28402	.95882	.29621	3.37594		30
	45	.28820	.95757	.30096	3.32264		15
17	0	.29237	.95630	.30573	3.27085	73	0
	15	.29654	.95502	.31051	3.22053		45
	30	.30070	.95372	.31530	3.17159		30
	45	.30486	.95240	.32010	3.12400		15
18	0	.30902	.95106	.32492	3.07768	72	0
	15	.31316	.94970	.32975	3.03260		45
	30	.31730	.94832	.33459	2.98868		30
	45	.32144	.94693	.33945	2.94591		15
19	0	.32557	.94552	.34433	2.90421	71	0
	15	.32969	.94409	.34921	2.86356		45
	30	.33381	.94264	.35412	2.82391		30
	45	.33792	.94118	.35904	2.78523		15
20	0	.34202	.93969	.36397	2.74748	70	0
	15	.34612	.93819	.36892	2.71062		45
	30	.35021	.93667	.37388	2.67462		30
	45	.35429	.93514	.37887	2.63945		15
21	0	.35837	.93358	.38386	2.60509	69	0
	15	.36244	.93201	.38888	2.57150		45
	30	.36650	.93042	.39391	2.53865		30
	45	.37056	.92881	.39896	2.50652		15
22	0	.37461	.92718	.40403	2.47509	68	0
	15	.37865	.92554	.40911	2.44433		45
	30	.38268	.92388	.41421	2.41421		30
	45	.38671	.92220	.41933	2.38473		15
23	0	.39073	.92050	.42447	2.35585	67	0
	15	.39474	.91879	.42963	2.32756		45
	30	.39875	.91706	.43481	2.29984		30
	45	.40275	.91531	.44001	2.27267		15
24	0	.40674	.91355	.44523	2.24604	66	0
	15	.41072	.91176	.45047	2.21992		45
	30	.41469	.90996	.45573	2.19430		30
	45	.41866	.90814	.46101	2.16917		15
25	0	.42262	.90631	.46631	2.14451	65	0
	15	.42657	.90446	.47163	2.12030		45
	30	.43051	.90259	.47697	2.09654		30
	45	.43445	.90070	.48234	2.07321		15
26	0	.43837	.89879	.48773	2.05030	64	0
	15	.44229	.89687	.49314	2.02780		45
	30	.44620	.89493	.49858	2.00569		30
	45	.45010	.89298	.50404	1.98396		15
27	0	.45399	.89101	.50952	1.96261	63	0
	15	.45787	.88902	.51503	1.94162		45
	30	.46175	.88701	.52057	1.92098		30
	45	.46561	.88499	.52612	1.90069		15
28	0	.46947	.88295	.53171	1.88073	62	0
	15	.47332	.88089	.53732	1.86109		45
	30	.47716	.87882	.54295	1.84177		30
	45	.48099	.87673	.54862	1.82276		15
29	0	.48481	.87462	.55431	1.80405	61	0
	15	.48862	.87250	.56003	1.78563		45
	30	.49242	.87036	.56577	1.76749		30
	45	.49622	.86820	.57155	1.74964		15
30	0	.50000	.86603	.57735	1.73205	60	0

TABLE II—(*Continued*)

D	*M*	Sines	Cosines	Tang.	Cotang.	*D*	*M*
30	0	.50000	.86603	.57735	1.73205	60	0
	15	.50377	.86384	.58318	1.71473		45
	30	.50754	.86163	.58904	1.69766		30
	45	.51129	.85941	.59494	1.68085		15
31	0	.51504	.85717	.60086	1.66428	59	0
	15	.51877	.85491	.60681	1.64795		45
	30	.52250	.85264	.61280	1.63185		30
	45	.52621	.85035	.61882	1.61598		15
32	0	.52992	.84805	.62487	1.60033	58	0
	15	.53361	.84573	.63095	1.58490		45
	30	.53730	.84339	.63707	1.56969		30
	45	.54097	.84104	.64322	1.55467		15
33	0	.54464	.83867	.64941	1.53986	57	0
	15	.54829	.83629	.65563	1.52525		45
	30	.55194	.83389	.66188	1.51084		30
	45	.55557	.83147	.66818	1.49661		15
34	0	.55919	.82904	.67451	1.48256	56	0
	15	.56280	.82659	.68087	1.46870		45
	30	.56641	.82413	.68728	1.45501		30
	45	.57000	.82165	.69372	1.44149		15
35	0	.57358	.81915	.70021	1.42815	55	0
	15	.57715	.81664	.70673	1.41497		45
	30	.58070	.81412	.71329	1.40195		30
	45	.58425	.81157	.71990	1.38909		15
36	0	.58779	.80902	.72654	1.37638	54	0
	15	.59131	.80644	.73323	1.36383		45
	30	.59482	.80386	.73996	1.35142		30
	45	.59832	.80125	.74673	1.33916		15
37	0	.60181	.79864	.75355	1.32704	53	0
	15	.60529	.79600	.76042	1.31507		45
	30	.60876	.79335	.76733	1.30323		30
	45	.61222	.79069	.77428	1.29152		15
38	0	.61566	.78801	.78129	1.27994	52	0
	15	.61909	.78532	.78834	1.26849		45
	30	.62251	.78261	.79543	1.25717		30
	45	.62592	.77988	.80258	1.24597		15
39	0	.62932	.77715	.80978	1.23490	51	0
	15	.63271	.77439	.81703	1.22394		45
	30	.63608	.77162	.82434	1.21310		30
	45	.63944	.76884	.83169	1.20237		15
40	0	.64279	.76604	.83910	1.19175	50	0
	15	.64612	.76323	.84656	1.18125		45
	30	.64945	.76041	.85408	1.17085		30
	45	.65276	.75756	.86165	1.16056		15
41	0	.65606	.75471	.86929	1.15037	49	0
	15	.65935	.75184	.87698	1.14029		45
	30	.66262	.74896	.88472	1.13029		30
	45	.66588	.74606	.89253	1.12041		15
42	0	.66913	.74314	.90040	1.11061	48	0
	15	.67237	.74022	.90834	1.10091		45
	30	.67559	.73728	.91633	1.09131		30
	45	.67880	.73432	.92439	1.08179		15
43	0	.68200	.73135	.93251	1.07237	47	0
	15	.68518	.72837	.94071	1.06303		45
	30	.68835	.72537	.94896	1.05378		30
	45	.69151	.72236	.95729	1.04461		15
44	0	.69466	.71934	.96569	1.03553	46	0
	15	.69779	.71630	.97416	1.02653		45
	30	.70091	.71325	.98270	1.01761		30
	45	.70401	.71019	.99131	1.00876		15
45	0	.70711	.70711	1.00000	1.00000	45	0

TABLE III

Common Logarithms

N	0	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774

TABLE III—(*Continued*)

N	0	1	2	3	4	5	6	7	8	9
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996

Specific Gravity of Materials

TABLE IV

Water	1.00	Copper	8.90
Kerosene	0.80	Silver	10.50
Oak	0.80	Lead	11.30
Gasoline	0.90	Mercury	13.60
Aluminum	2.58	Tungsten	18.80
Tin	7.29	Gold	19.30
Iron	7.40	Platinum	21.50

TABLE V
Weight and Specific Gravity of Liquids

Liquids at 32° F.	Weight of 1 Cu. Ft. in Lbs.	Weight of 1 Gal. (Imp.) in Lbs.	Specific Gravity Water = 1.000
Mercury	848.7	136.0	13.596
Bromine	185.1	29.7	2.966
Sulphuric acid	114.9	18.4	1.84
Nitrous acid	96.8	15.5	1.55
Chloroform	95.5	15.3	1.53
Water of the Dead Sea	77.4	12.4	1.24
Nitric acid	76.2	12.2	1.22
Acetic acid	67.4	10.8	1.08
Milk	64.3	10.3	1.03
Sea water	64.05	10.3	1.026
Pure water (distilled) at 39° F.	62.425	10.0	1.0
Oil, linseed	58.7	9.4	0.94
Oil, whale	57.4	9.2	0.92
Oil, turpentine	54.3	8.7	0.87
Petroleum	54.9	8.8	0.88
Naphtha	53.1	8.5	0.85
Ether, nitric	69.3	11.1	1.11
Ether, sulphurous	67.4	10.8	1.08
Ether, acetic	55.6	8.9	0.89
Ether, hydrochloric	54.3	8.7	0.87
Ether, sulphuric	44.9	7.2	0.72
Alcohol, proof spirit	57.4	9.2	0.92
Alcohol, pure	49.3	7.9	0.79
Benzine	53.1	8.5	0.85

TABLE VI
Melting Point of Materials

Mercury	38° F.	Manganese	2300° F.
Tin	450	Steel	2500
Lead	621	Silicon	2588
Zinc	787	Nickel	2646
Antimony	1166	Cobalt	2696
Aluminum	1218	Wrought iron	2900
Radium	1292	Vanadium	3191
Barium	1562	Platinum	3200
Bronze	1675	Titanium	3272
Silver	1762	Uranium	3362
Gold	1945	Molybdenum	4500
Copper	1981	Tungsten	5430
Cast iron	2300	Carbon	6500

TABLE VII

Strength of Miscellaneoua Metala

Material	Modulus of Rigidity	Elastic Limit	Tensile Strength
Phosphor bronze	13,000,000	20,000	30,000
Manganese bronze	15,000,000	35,000	50,000
Aluminum bronze	15,000,000	50,000	75,000
Wrought iron	28,000,000	30,000	40,000
Mild open hearth steel	28,000,000	40,000	60,000
Tool steel	29,000,000	80,000	110,000
Nickel steel	28,000,000	80,000	110,000
Nickel chrome steel	30,000,000	160,000	180,000
Vanadium steel treated	30,000,000	220,000	228,000

TABLE VIII

Tapers and Angles

Taper per Foot	Included <		With Center Line ′		Taper per Inch	Taper per Inch from Center Line
	Deg.	Min.	Deg.	Min.		
$\frac{1}{8}$	0	36	0	18	.010416	.005203
$\frac{3}{16}$	0	54	0	27	.015625	.007812
$\frac{1}{4}$	1	12	0	36	.020833	.010416
$\frac{5}{16}$	1	30	0	45	.026042	.013021
$\frac{3}{8}$	1	47	0	53	.031250	.015625
$\frac{7}{16}$	2	05	1	02	.036458	.018229
$\frac{1}{2}$	2	23	1	11	.041667	.020833
$\frac{9}{16}$	2	42	1	21	.046875	.023438
$\frac{5}{8}$	3	00	1	30	.052084	.026042
$\frac{11}{16}$	3	18	1	39	.057292	.028646
$\frac{3}{4}$	3	25	1	47	.062500	.031250
$\frac{13}{16}$	3	52	1	56	.067708	.033854
$\frac{7}{8}$	4	12	2	06	.072917	.036456
$\frac{15}{16}$	4	28	2	14	.078125	.039063
1	4	45	2	23	.083330	.041667
$1\frac{1}{4}$	5	58	2	59	.104666	.052084
$1\frac{1}{2}$	7	08	3	34	.125000	.062500
$1\frac{3}{4}$	8	20	4	10	.145833	.072917
2	9	32	4	46	.166666	.083332
$2\frac{1}{2}$	11	54	5	57	.208333	.104166
3	14	16	7	08	.250000	.125000
$3\frac{1}{2}$	16	36	8	18	.291666	.145833
4	18	54	9	27	.333333	.166666
$4\frac{1}{2}$	21	40	10	50	.375000	.187500
5	24	04	12	02	.416666	.208333
6	28	06	14	03	.500000	.250000

Brown and Sharp Standard Tapera

No. of Taper,	4	5	7	9
Diameter at small end,	.35 in.	.45 in.	.60 in.	.90 in.

This taper is ½″ per foot.

Morse Standard Tapers

No. of Taper,	1	2	3	4
Diameter at small end,	.37 in.	.57 in.	.78 in.	1.02 in.

This taper is about ⅝″ per foot.

TABLE IX

Table of Speeds

Diam. Ins.	Cutting Speeds in Feet per Minute								
	20	30	40	50	60	70	80	90	100
	Revolutions per Minute								
¼	306	458	611	764	916	1070	1222	1376	1528
⅜	204	306	407	509	612	712	814	916	1019
½	153	229	306	382	458	534	612	688	764
⅝	122	183	244	306	366	428	488	550	611
¾	102	153	204	255	306	356	408	458	509
⅞	87	131	175	218	262	306	350	392	437
1	76	115	153	191	230	268	306	344	382
1⅛	68	102	136	170	204	238	272	306	340
1¼	61	92	122	153	184	214	244	274	306
1⅜	56	83	111	139	167	194	222	250	278
1½	51	76	102	127	152	178	204	228	255
1⅝	47	71	94	118	141	165	188	212	235
1¾	44	65	87	109	130	152	174	196	218
1⅞	41	61	82	102	122	143	163	183	204
2	38	57	76	95	114	134	152	172	191
2⅛	36	54	72	90	108	126	144	162	180
2¼	34	51	68	85	102	119	136	153	170
2⅜	32	48	64	80	97	112	129	145	161
2½	31	46	61	76	92	106	122	134	153
2⅝	29	44	58	73	88	102	117	130	146
3¾	28	42	56	70	83	97	111	125	139
2⅞	27	40	53	67	80	93	106	119	133
3	25	38	51	64	76	90	102	114	127

Diam. Inches	Cutting Speeds in Feet per Minute							
	110	120	130	140	150	160	170	180
	Revolutions per Minute							
$\frac{1}{4}$	1681	1833	1986	2139	2292	2462	2615	2780
$\frac{3}{8}$	1120	1222	1324	1426	1528	1632	1735	1836
$\frac{1}{2}$	840	917	993	1070	1146	1221	1298	1374
$\frac{5}{8}$	672	733	794	856	917	976	1036	1098
$\frac{3}{4}$	560	611	662	713	764	816	867	918
$\frac{7}{8}$	480	524	568	611	655	699	742	786
1	420	458	497	535	573	611	649	687
$1\frac{1}{8}$	373	407	441	475	509	542	576	610
$1\frac{1}{4}$	336	367	397	428	458	489	520	551
$1\frac{3}{8}$	306	333	361	389	417	444	472	500
$1\frac{1}{2}$	280	306	331	357	382	407	433	458
$1\frac{5}{8}$	259	282	306	329	353	377	400	423
$1\frac{3}{4}$	240	262	284	306	327	349	371	393
$1\frac{7}{8}$	224	244	265	285	306	326	346	366
2	210	229	248	267	287	306	324	344
$2\frac{1}{8}$	198	216	234	252	270	288	306	323
$2\frac{1}{4}$	187	204	221	238	255	272	289	306
$2\frac{3}{8}$	177	193	210	225	241	257	273	290
$2\frac{1}{2}$	168	183	199	214	229	244	260	275
$2\frac{5}{8}$	160	175	189	204	218	233	248	262
$2\frac{3}{4}$	153	167	181	194	208	222	236	250
$2\frac{7}{8}$	146	159	173	186	199	213	226	239
3	140	153	166	178	191	204	216	229

TABLE X

Weights and Areas of Round, Square and Hexagon Steel

Weight of one cubic inch = .2836 lbs.
Weight of one cubic foot = 490 lbs.

Thickness or Diameter	Area = Diam. ^{2}x .7854			Area = Side ^{2}x 1		Area = Diam. ^{2}x .866	
	Round			Square		Hexagon	
	Weight Per Inch	Area Square Inches	Circumference Inches	Weight Per Inch	Area Square Inches	Weight Per Inch	Area Square Inches
$\frac{1}{32}$	.0002	.0008	.0981	.0003	.0010	.0002	.0008
$\frac{1}{16}$	.0009	.0031	.1963	.0011	.0039	.0010	.0034
$\frac{3}{32}$	.0020	.0069	.2995	.0025	.0088	.0022	.0076
$\frac{1}{8}$	.0035	.0123	.3927	.0044	.0156	.0038	.0135
$\frac{5}{32}$	.0054	.0192	.4908	.0069	.0244	.0060	.0211
$\frac{3}{16}$	.0078	.0276	.5890	.0101	.0352	.0086	.0304
$\frac{7}{32}$	.0107	.0376	.6872	.0136	.0479	.0118	.0414
$\frac{1}{4}$	.0139	.0491	.7854	.0177	.0625	.0154	.0540
$\frac{9}{32}$	.0176	.0621	.8835	.0224	.0791	.0194	.0686
$\frac{5}{16}$	.0218	.0767	.9817	.0277	.0977	.0240	.0846
$\frac{11}{32}$	.0263	.0928	1.0799	.0335	.1182	.0290	.1023
$\frac{3}{8}$	.0313	.1104	1.1781	.0405	.1406	.0345	.1218
$\frac{13}{32}$	.0368	.1296	1.2762	.0466	.1651	.0405	.1428
$\frac{7}{16}$	.0426	.1503	1.3744	.0543	.1914	.0470	.1658
$\frac{15}{32}$	.0489	.1726	1.4726	.0623	.2197	.0540	.1903
$\frac{1}{2}$	.0557	.1963	1.5708	.0709	.2500	.0614	.2161
$\frac{17}{32}$	.0629	.2217	1.6689	.0800	.2822	.0693	.2444
$\frac{9}{16}$	.0705	.2485	1.7671	.0897	.3164	.0777	.2743
$\frac{19}{32}$	.0785	.2769	1.8653	.1036	.3526	.0866	.3053
$\frac{5}{8}$	.0870	.3068	1.9635	.1108	.3906	.0959	.3383
$\frac{21}{32}$	.0959	.3382	2.0616	.1221	.4307	.1058	.3730
$\frac{11}{16}$	.1053	.3712	2.1598	.1340	.4727	.1161	.4093
$\frac{23}{32}$	.1151	.4057	2.2580	.1465	.5166	.1270	.4474
$\frac{3}{4}$	.1253	.4418	2.3562	.1622	.5625	.1382	.4871
$\frac{25}{32}$	.1359	.4794	2.4543	.1732	.6103	.1499	.5286
$\frac{13}{16}$	.1470	.5185	2.5525	.1872	.6602	.1620	.5712
$\frac{27}{32}$	.1586	.5591	2.6507	.2019	.7119	.1749	.6165
$\frac{7}{8}$	.1705	.6013	2.7489	.2171	.7656	.1880	.6631
$\frac{29}{32}$	.1829	.6450	2.8470	.2329	.8213	.2015	.7112
$\frac{15}{16}$	.1958	.6903	2.9452	.2492	.8789	.2159	.7612
$\frac{31}{32}$	.2090	.7371	3.0434	.2661	.9384	.2305	.8127
1	.2227	.7854	3.1416	.2683	1.0000	.2456	.8643

TABLE X—(*Continued*)

Thickness or Diameter	Area = Diam.^{2}x 7854			Area = Side ^{2}x 1		Area = Diam. ^{2}x .866	
	Round			Square		Hexagon	
	Weight Per Inch	Area Square Inches	Circumference Inches	Weight Per Inch	Area Square Inches	Weight Per Inch	Area Square Inches
$1\frac{1}{16}$	.2515	.8866	3.3379	.3201	1.1289	.2773	.9776
$1\frac{1}{8}$	.2819	.9940	3.5343	.3589	1.2656	.3109	1.0973
$1\frac{3}{18}$	.3141	1.1075	3.7306	.4142	1.4102	.3464	1.2212
$1\frac{1}{4}$	.3480	1.2272	3.9270	.4431	.15625	.3838	1.3531
$1\frac{5}{16}$	.3837	1.3530	4.1233	.4885	1.7227	.4231	1.4919
$1\frac{3}{8}$	.4211	1.4849	4.3197	.5362	1.8906	.4643	1.6373
$1\frac{7}{16}$	.4603	.16230	4.5160	.5860	2.0664	.5076	1.7898
$1\frac{1}{2}$	.5012	1.7671	4.7124	.6487	2.2500	.5526	1.9485
$1\frac{9}{16}$	.5438	1.9175	4.9087	.6930	2.4414	.5996	2.1143
$1\frac{5}{8}$	.5882	2.0739	5.1051	.7489	2.6406	.6480	2.2847
$1\frac{11}{16}$	.6343	2.2365	5.3014	.8076	2.8477	.6994	2.4662
$1\frac{3}{4}$	.6821	2.4053	5.4978	.8685	3.0625	.7521	2.6522
$1\frac{13}{16}$	.7317	2.5802	5.6941	.9316	3.2852	.8069	2.8450
$1\frac{7}{8}$	.7831	2.7612	5.8905	.9970	3.5156	.8635	3.0446
$1\frac{15}{16}$	.8361	2.9483	6.0868	1.0646	3.7539	.9220	3.2509
2	.8910	3.1416	6.2832	1.1342	4.0000	.9825	3.4573
$2\frac{1}{16}$	.9475	3.3410	6.4795	1.2064	4.2539	1.0448	3.6840
$2\frac{1}{8}$	1.0058	3.5466	6.6759	1.2806	4.5156	1.1091	3.9106
$2\frac{3}{16}$	1.0658	3.7583	6.8722	1.3570	4.7852	1.1753	4.1440
$2\frac{1}{4}$	1.1276	3.9761	7.0686	1.4357	5.0625	1.2434	4.3892
$2\frac{5}{16}$	1.1911	4.2000	7.2649	1.5165	5.3477	1.3135	4.6312
$2\frac{3}{8}$	1.2564	4.4301	7.4613	1.6569	5.6406	1.3854	4.8849
$2\frac{7}{18}$	1.3234	4.6664	7.6575	1.6849	5.9414	1.4593	5.1454
$2\frac{1}{2}$	1.3921	4.9087	7.8540	1.7724	6.2500	1.5351	5.4126
$2\frac{5}{8}$	1.5348	5.4119	8.2467	1.9541	6.8906	1.6924	5.9674
$2\frac{3}{4}$	1.6845	5.9396	8.6394	2.1446	7.5625	1.8574	6.5493
$2\frac{7}{8}$	1.8411	6.4918	9.0321	2.3441	8.2656	2.0304	7.1590
3	2.0046	7.0686	9.4248	2.5548	9.0000	2.2105	7.7941
$3\frac{1}{8}$	2.1752	7.6699	9.8175	2.7719	9.7656	2.3986	8.4573
$3\frac{1}{4}$	2.3527	8.2958	10.2102	2.9954	10.5625	2.5918	9.1387
$3\frac{3}{8}$	2.5371	8.9462	10.6029	3.2303	11.3906	2.7977	9.8646
$3\frac{1}{2}$	2.7286	9.6211	10.9956	3.4740	12.2500	3.0083	10.6089

TABLF X—(*Continued*)

Thickness or Diameter	Area = Diam. ^{2}x .7854			Area = Side ^{2}x 1		Area = Diam. ^{2}x .866	
	Round			Square		Hexagon	
	Weight Per Inch	Area Square Inches	Circum-ference Inches	Weight Per Inch	Area Square Inches	Weight Per Inch	Area Square Inches
$3\frac{5}{8}$	2.9269	10.3206	11.3883	3.7265	13.1407	3.2275	11.3798
$3\frac{3}{4}$	3.1323	11.0447	11.7810	3.9880	14.0625	3.4539	12.1785
$3\frac{7}{8}$	3.3446	11.7932	12.1737	4.2582	15.0156	3.6880	13.0035
4	3.5638	12.5664	12.5664	4.5374	16.0000	3.9298	13.8292
$4\frac{1}{8}$	3.7900	13.3640	12.9591	4.8254	17.0156	4.1792	14.7359
$4\frac{1}{4}$	4.0232	14.1863	13.3518	5.1223	18.0625	4.4364	15.6424
$4\frac{3}{8}$	4.2634	15.0332	13.7445	5.4280	19.1406	4.7011	16.5761
$4\frac{1}{2}$	4.5105	15.9043	14.1372	5.7426	20.2500	4.9736	17.5569
$4\frac{5}{8}$	4.7645	16.8002	14.5299	6.0662	21.3906	5.2538	18.5249
$4\frac{3}{4}$	5.0255	17.7205	14.9226	6.6276	22.5625	5.5416	19.5397
$4\frac{7}{8}$	5.2935	18.6655	15.3153	6.7397	23.7656	5.8371	20.5816
5	5.5685	19.6350	15.7080	7.0897	25.0000	6.1403	21.6503
$5\frac{1}{8}$	5.8504	20.6290	16.1007	7.4496	26.2656	6.4511	22.7456
$5\frac{1}{4}$	6.1392	21.6475	16.4934	7.8164	27.5624	6.7697	23.8696
$5\frac{3}{8}$	6.4351	22.6905	16.8861	8.1930	28.8906	7.0959	25.0198
$5\frac{1}{2}$	6.7379	23.7583	17.2788	8.5786	30.2500	7.4298	26.1971
$5\frac{5}{8}$	7.0476	24.8505	17.6715	8.9729	31.6406	7.7713	27.4013
$5\frac{3}{4}$	7.3643	25.9672	18.0642	9.3762	33.0625	8.1214	28.6361
$5\frac{7}{8}$	7.6880	27.1085	18.4569	9.7883	34.5156	8.4774	29.8913
6	8.0186	28.2743	18.8496	10.2192	36.0000	8.8420	31.1765
$6\frac{1}{4}$	8.7007	30.6796	19.6350	11.0877	39.0625	9.5943	33.8291
$6\frac{1}{2}$	9.4107	33.1831	20.4204	11.9817	42.2500	10.3673	36.5547
$6\frac{3}{4}$	10.1485	35.7847	21.2058	12.9211	45.5625	11.1908	39.4584
7	10.9142	38.4845	21.9912	13.8960	49.0000	12.0351	42.4354
$7\frac{1}{2}$	12.5291	44.1786	23.5620	15.9520	56.2500	13.8158	48.7142
8	14.2553	50.2655	25.1328	18.1497	64.0000	15.7192	55.3169

Multiply above weights by .993 for wrought iron, .918 for cast iron 1.0331 for cast brass, 1.1209 for copper, 1.1748 for phos. bronze, and .3265 for aluminum.

TABLE XI

Circumference and Area of Circles (1/16″ to 2″)

Diam.	Circum.	Area	Diam.	Circum.	Area
$\frac{1}{16}$	0.19635	0.00307	$1\frac{1}{16}$	3.3379	0.8866
$\frac{1}{8}$	0.39270	0.01227	$1\frac{1}{8}$	3.5343	0.9940
$\frac{3}{16}$	0.58905	0.02761	$1\frac{3}{16}$	3.7306	1.1075
$\frac{1}{4}$	0.78540	0.04909	$1\frac{1}{4}$	3.9270	1.2272
$\frac{5}{16}$	0.98175	0.07670	$1\frac{5}{16}$	4.1233	1.3530
$\frac{3}{8}$	1.1781	0.11045	$1\frac{3}{8}$	4.3197	1.4849
$\frac{7}{16}$	1.3744	0.15033	$1\frac{7}{16}$	4.5160	1.6230
$\frac{1}{2}$	1.5708	0.19635	$1\frac{1}{2}$	4.7124	1.7671
$\frac{9}{16}$	1.7671	0.24850	$1\frac{9}{16}$	4.9087	1.9175
$\frac{5}{8}$	1.9635	0.30680	$1\frac{5}{8}$	5.1051	2.0739
$\frac{11}{16}$	2.1598	0.37122	$1\frac{11}{16}$	5.3014	2.2365
$\frac{3}{4}$	2.3562	0.44179	$1\frac{3}{4}$	5.4978	2.4053
$\frac{13}{16}$	2.5525	0.51849	$1\frac{13}{16}$	5.6941	2.5802
$\frac{7}{8}$	2.7489	0.60132	$1\frac{7}{8}$	5.8905	2.7612
$\frac{15}{16}$	2.9452	0.69029	$1\frac{15}{16}$	6.0868	2.9483
1	3.1416	0.7854	2	6.2832	3.1416

TABLE XII

Standard Dimensions of Wrought Iron Welded Tubes.

Briggs Standard

Diameter of Tube			Thickness of Metal	Threaded End	
Nominal Inside	Actual Inside	Actual Outside		Number of Threads per Inch	Length of Perfect Thread
$\frac{1}{8}$ in.	0.270 in.	0.405 in.	0.068 in.	27	0.19 in.
$\frac{1}{4}$ in.	0.364 in.	0.540 in.	0.088 in.	18	0.29 in.
$\frac{3}{8}$ in.	0.494 in.	0.675 in.	0.091 in.	18	0.30 in.
$\frac{1}{2}$ in.	0.623 in.	0.840 in.	0.109 in.	14	0.39 in.
$\frac{3}{4}$ in.	0.824 in.	1.050 in.	0.113 in.	14	0.40 in.
1 in.	1.048 in.	1.315 in.	0.134 in.	$11\frac{1}{2}$	0.51 in.
$1\frac{1}{4}$ in.	1.380 in.	1.660 in.	0.140 in.	$11\frac{1}{2}$	0.54 in.
$1\frac{1}{2}$ in.	1.610 in.	1.900 in.	0.145 in.	$11\frac{1}{2}$	0.55 in.
2 in.	2.067 in.	2.375 in.	0.154 in.	$11\frac{1}{2}$	0.58 in.
$2\frac{1}{2}$ in.	2.468 in.	2.875 in.	0.204 in.	8	0.89 in.
3 in.	3.067 in.	3.500 in.	0.217 in.	8	0.95 in.
$3\frac{1}{2}$ in.	3.548 in.	4.000 in.	0.226 in.	8	1.00 in.
4 in.	4.026 in.	4.500 in.	0.237 in.	8	1.05 in.
$4\frac{1}{2}$ in.	4.508 in.	5.000 in.	0.246 in.	8	1.10 in.
5 in.	5.045 in.	5.563 in.	0.259 in.	8	1.16 in.
6 in.	6.065 in.	6.625 in.	0.280 in.	8	1.26 in.
7 in.	7.023 in.	7.625 in.	0.301 in.	8	1.36 in.
8 in.	7.982 in.	8.625 in.	0.322 in.	8	1.46 in.
9 in.	9.000 in.	9.688 in.	0.344 in.	8	1.57 in.
10 in.	10.019 in.	10.750 in.	0.366 in.	8	1.68 in.

TABLE XII—(*Continued*)

The Sizes of Twist Drilla to Be Used in Drilling Holes to Be Reamed with Pipe Reamer, and Threaded with Pipe Tap, are aa Follows:

Size Tap	Diameter Drill	Size Tap	Diameter Drill
$\frac{1}{8}$ inch	$\frac{21}{64}$ inch	1 inch	$1\frac{3}{16}$ inch
$\frac{1}{4}$ inch	$\frac{29}{64}$ inch	$1\frac{1}{4}$ inch	$1\frac{15}{32}$ inch
$\frac{3}{8}$ inch	$\frac{19}{32}$ inch	$1\frac{1}{2}$ inch	$1\frac{23}{32}$ inch
$\frac{1}{2}$ inch	$\frac{23}{32}$ inch	2 inch	$2\frac{3}{16}$ inch
$\frac{3}{4}$ inch	$\frac{15}{16}$ inch	$2\frac{1}{2}$ inch	$2\frac{11}{16}$ inch
		3 inch	$3\frac{5}{16}$ inch

The Word Diameter, when Used for Gaa or Steam Pipe Measure, Always Means Inside Diameter

TABLE XIII

Sizes of Tap Drills for Taps with U. S. Standard Thread

Size of Tap	No. of Threads per Inch	Size of Drill	Size of Tap	No. of Threads per Inch	Size of Drill
$\frac{1}{4}$	20	$\frac{3}{16}$	$1\frac{1}{4}$	7	$1\frac{5}{64}$
$\frac{5}{16}$	18	C	$1\frac{3}{8}$	6	$1\frac{11}{64}$
$\frac{3}{8}$	16	N	$1\frac{1}{2}$	6	$1\frac{19}{64}$
$\frac{7}{16}$	14	S	$1\frac{5}{8}$	$5\frac{1}{2}$	$1\frac{25}{64}$
$\frac{1}{2}$	13	$\frac{13}{32}$	$1\frac{3}{4}$	5	$1\frac{1}{2}$
$\frac{9}{16}$	12	$\frac{29}{64}$	$1\frac{7}{8}$	5	$1\frac{5}{8}$
$\frac{5}{8}$	11	$\frac{33}{64}$	2	$4\frac{1}{2}$	$1\frac{23}{32}$
$\frac{11}{16}$	11	$\frac{37}{64}$	$2\frac{1}{8}$	$4\frac{1}{2}$	$1\frac{27}{32}$
$\frac{3}{4}$	10	$\frac{5}{8}$	$2\frac{1}{4}$	$4\frac{1}{2}$	$1\frac{31}{32}$
$\frac{13}{16}$	10	$\frac{11}{16}$	$2\frac{3}{8}$	4	$2\frac{1}{16}$
$\frac{7}{8}$	9	$\frac{47}{64}$	$2\frac{1}{2}$	4	$2\frac{3}{16}$
$\frac{15}{16}$	9	$\frac{51}{64}$			
1	8	$\frac{27}{32}$			
$1\frac{1}{8}$	7	$\frac{61}{64}$			

TABLE XIV

Decimal Equivalents of the Number of Twist Drills and Steel Wire Gage

No.	Size of No. in Decimals	No.	Size of No. in Decimals	No.	Size of No. in Decimals	No.	Size of No. in Decimals
1	.2280	21	.1590	41	.0960	61	.0390
2	.2210	22	.1570	42	.0935	62	.0380
3	.2130	23	.1540	43	.0890	63	.0370
4	.2090	24	.1520	44	.0860	64	.0360
5	.2055	25	.1495	45	.0820	65	.0350
6	.2040	26	.1470	46	.0810	66	.0330
7	.2010	27	.1440	47	.0785	67	.0320
8	.1990	28	.1405	48	.0760	68	.0310
9	.1960	29	.1360	49	.0730	69	.02925
10	.1935	30	.1285	50	.0700	70	.0280
11	.1910	31	.1200	51	.0670	71	.0260
12	.1890	32	.1160	52	.0635	72	.0250
13	.1850	33	.1130	53	.0595	73	.0240
14	.1820	34	.1110	54	.0550	74	.0225
15	.1800	35	.1100	55	.0520	75	.0210
16	.1770	36	.1065	56	.0465	76	.0200
17	.1730	37	.1040	57	.0430	77	.0180
18	.1695	38	.1015	58	.0420	78	.0160
19	.1660	39	.0995	59	.0410	79	.0145
20	.1610	40	.0980	60	.0400	80	.0135

Letter Sizes

A.......234	H.......266	O.......316	U.......368
B.......238	I.......272	P.......323	V.......377
C.......242	J.......277	Q.......331	W.......386
D.......246	K.......281	R.......339	X.......397
E.......25	L.......29	S.......348	Y.......404
F.......257	M.......295	T.......358	Z.......413
G.......261	N.......302		

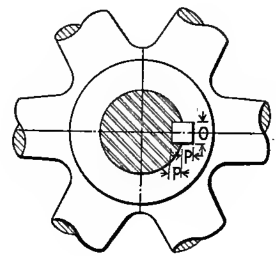


TABLE XV

Standard Key Seats

Diameter of Hole, Inches	O Width of Key Seat	P Depth of Key Seat	Diameter of Hole, Inches	O Width of Key Seat	P Depth of Key Seat
$\frac{3}{8}$	$\frac{3}{32}$	$\frac{3}{64}$	$3\frac{1}{4}$ to $3\frac{11}{16}$	$\frac{7}{8}$	$\frac{7}{16}$
$\frac{7}{16}$ to $\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{16}$	$3\frac{3}{4}$ to $4\frac{3}{16}$	1	$\frac{1}{2}$
$\frac{9}{16}$ to $\frac{5}{8}$	$\frac{5}{32}$	$\frac{5}{64}$	$4\frac{1}{4}$ to $4\frac{11}{16}$	$1\frac{1}{8}$	$\frac{9}{16}$
$\frac{11}{16}$ to $\frac{3}{4}$	$\frac{3}{16}$	$\frac{3}{32}$	$4\frac{3}{4}$ to $5\frac{3}{16}$	$1\frac{1}{4}$	$\frac{5}{8}$
$\frac{13}{16}$ to $\frac{7}{8}$	$\frac{7}{32}$	$\frac{7}{64}$	$5\frac{1}{4}$ to $5\frac{11}{16}$	$1\frac{3}{8}$	$\frac{11}{16}$
$\frac{15}{16}$ to $1\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$	$5\frac{3}{4}$ to $6\frac{3}{16}$	$1\frac{1}{2}$	$\frac{3}{4}$
$1\frac{3}{16}$ to $1\frac{3}{8}$	$\frac{5}{16}$	$\frac{5}{32}$	$6\frac{1}{4}$ to $6\frac{11}{16}$	$1\frac{5}{8}$	$\frac{13}{16}$
$1\frac{7}{16}$ to $1\frac{5}{8}$	$\frac{3}{8}$	$\frac{3}{16}$	$6\frac{3}{4}$ to $7\frac{3}{16}$	$1\frac{3}{4}$	$\frac{7}{8}$
$1\frac{11}{16}$ to $1\frac{7}{8}$	$\frac{7}{16}$	$\frac{7}{32}$	$7\frac{1}{4}$ to $7\frac{11}{16}$	$1\frac{7}{8}$	$\frac{15}{16}$
$1\frac{15}{16}$ to $2\frac{1}{8}$	$\frac{1}{2}$	$\frac{1}{4}$	$7\frac{3}{4}$ to $8\frac{3}{16}$	2	1
$2\frac{3}{16}$ to $2\frac{11}{16}$	$\frac{5}{8}$	$\frac{5}{16}$	$8\frac{1}{4}$ to $8\frac{15}{16}$	$2\frac{1}{8}$	$1\frac{1}{16}$
$2\frac{3}{4}$ to $3\frac{3}{16}$	$\frac{3}{4}$	$\frac{3}{8}$	9 to 10	$2\frac{1}{4}$	$1\frac{1}{8}$

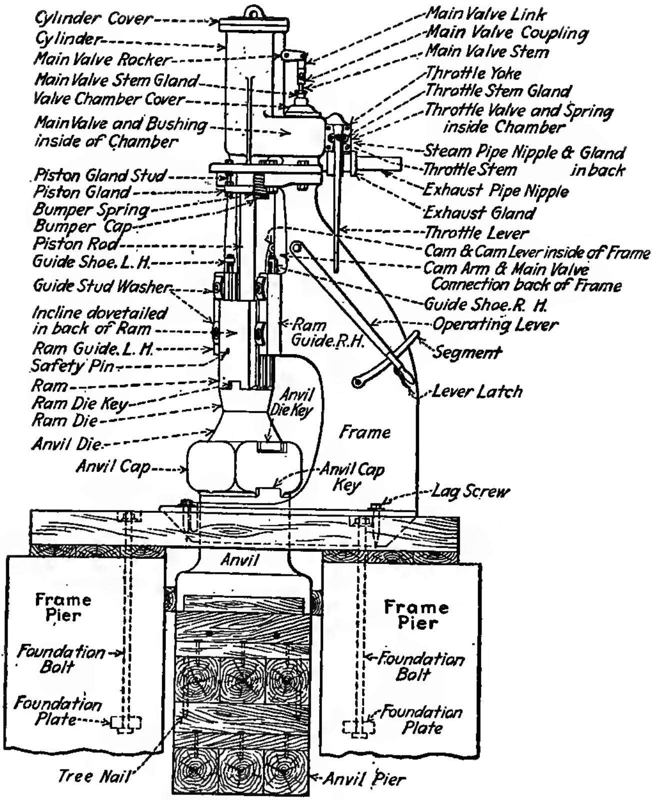
TABLE XVI

Multiplication Tables 25 × 20

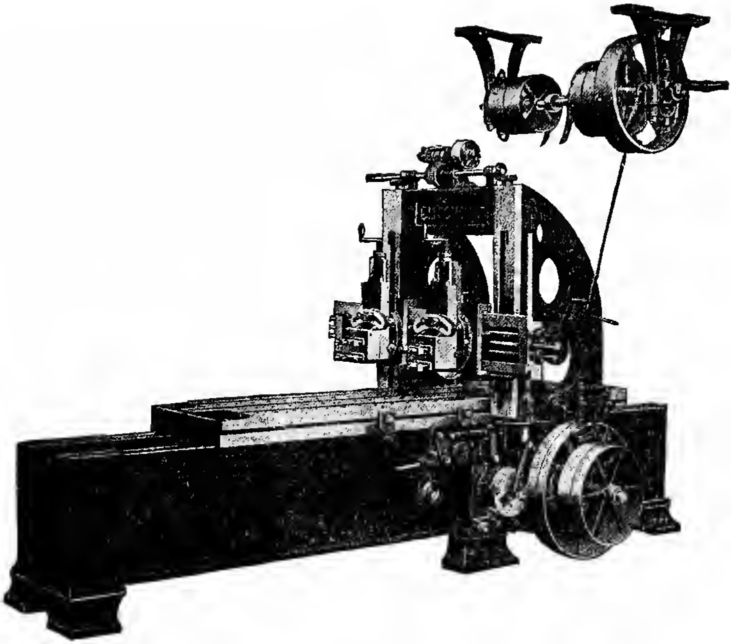
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400
21	42	63	84	105	126	147	168	189	210	231	252	273	294	315	336	357	378	399	420
22	44	66	88	110	132	154	176	198	220	242	264	286	308	330	352	374	396	418	440
23	46	69	92	115	138	161	184	207	230	253	276	299	322	345	368	391	414	437	460
24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	384	408	432	456	480
25	50	75	100	125	150	175	200	225	250	275	300	325	350	375	400	425	450	475	500

50 100 200 300 400

50 100 200 300 400 500

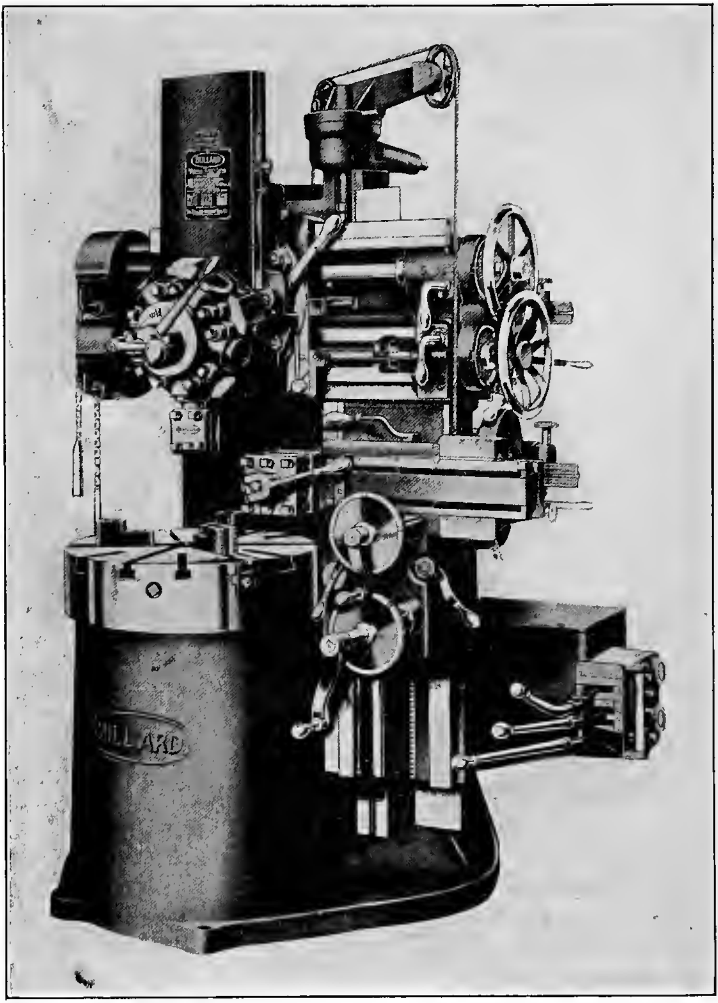


STEAM HAMMER



PLANER

A—Bed.
B—Reversing lever.
C—Feed friction.
D—Belt shifting mechanism.
E—Hand cross feed lever.
F—Vertical feed levers.
G—Tool head.
H—Cross rail.
I—Table or platen.
J—Reverse dog.



Boring Mill

ANSWERS

ANSWERS TO MATHEMATICS

Notation and Numeration—Page 3

5. 85.
8. 1, 648, 432.

Addition, Subtraction, Multiplication and Division—Page 8

1. (*a*) 880,369.	(*b*) 4,866.
2. (*a*) 32,983.	(*b*) 5,997.
3. (*a*) 774.	(*b*) 60,727.
4. (*a*) 86,081.	(*b*) 342.
5. (*a*) 35.	(*b*) 339.
6. (*a*) 9,990.	(*b*) 1,259.
7. (*a*) 7,056.	(*b*) 2,091.
8. (*a*) 22,960.	(*b*) 98,000.
9. (*a*) 1,566,585.	(*b*) 8,104,320.
10. (*a*) 843.	(*b*) 162.
11. (*a*) 23.	(*b*) 64,305.
12. (*a*) 10,770.	(*b*) 1,009.

Cancellation and Least Common Multiple—Page 9

1. 120.
2. 280.
3. 24.

4. (*a*) 252.	(*b*) 48.
5. (*a*) 60.	(*b*) 81.
6. (*a*) 72.	(*b*) 90.
7. (*a*) 60.	(*b*) 180.

Common Fractions—Page 13

1. $26\frac{2}{3}$; 3 1/7; $8\frac{1}{2}$; $6\frac{1}{3}$; $27\frac{1}{4}$; 2 6/83; 25 1/5; 1 131/763; 1 2/121; 1 1/12.
2. 21/4; 73/9; 20/3; 103/8; 127/9; 55/8; 143/12; 2449/49; 790320/889; 2134/17.
3. 1/6; 9/25; 1/18; 4/15; 5/12; 3/8; 5/8; 40/323; 47069/67598; 3 2/143.
4. 1 61/63; 1 23/352; 1 5/76; 224/225; 9/16; 1/20; 1/4; 14/15; 1 1/3.

5. 28/4; 36/2; 36/3; 96/12; 50/10; 18/3; 32/8; 42/7; 20/5; 221/13.
6. 6/7; 1/3; 31/32; 6 1/3; 14 1/4; 18 1/2; 124 30/37; 5 1/144; 14 3/7.
7. 40/64; 32/48; 12/128; 60/64; 65/160; 12/8; 200/24; 39/6.
8. 12/24, 8/24, 9/24; 160/180, 135/180, 72/180; 8 1/8, 3 5/8, 7/8; 1/16, 5 16/48; 32/36, 9/36, 2 18/36; 5 2/16, 3 4/16, 1/16; 24/64, 5 1/64, 28/64; 8 8/32, 2 16/32, 9/32; 8 32/64, 9/64.
9. (a) 8 19/24. (b) 48 13/24.
10. (a) 865 11/24. (b) 82 135/182.
11. (a) 88 389/420. (b) 512 37/48.
12. (a) 7 3/8. (b) 5/6.
13. (a) 6 17/24. (b) 61/64.
14. (a) 25 37/39. (b) 9761 7/8.
15. (a) 4/15. (b) 2 11/32.
16. (a) 1/4. (b) 182 13951/16384.
17. (a) 14 21/128. (b) 4670 13/40.
18. (a) 1 1/6. (b) 2.
19. (a) 1/2. (b) 4 127/158.
20. (a) 60 3/5. (b) 5 445/1868.
21. 8 23/32 ins.
22. 2 43/64 ins.
23. 6 1/4 hrs.
24. 3/8 ft.
25. $19.35.
26. 10 5/8 ft.
27. 4/45.
28. 3 21/22.
29. 145.
30. 9.

Decimal Fractions—Page 18

1. (a) 51,497.50. (b) 86,086.3.
2. (a) 126.375. (b) 46,494.38793.
3. (a) 2,749.3709467. (b) 1064.62638.
4. (a) 79.375. (b) 0.13503.
5. (a) 0.539991. (b) 0.058483.
6. (a) 0.611. (b) 273.397.
7. (a) 124.74. (b) 13,777.44.
8. (a) 697.5. (b) 0.6889.
9. (a) 0.46875. (b) 133.438.

10. (a) 1.820. (b) 10.762.
11. (a) 1,000. (b) 140,005,000.
12. (a) 0.0002. (b) 1.605.
13. (a) 0.0156; 0.125. (b) 0.7143; 0.375.
14. (a) 0.1094; 0.8889. (b) 0.6667; 4.8333.
15. (a) 1/2; 3/4. (b) 2 5/8; 4 13/32.
16. (a) 3 35/64; 7 3/16. (b) 25 3/64; 1 3/8.
17. $7.525.
18. $3.75.
19. $0.325.
20. 8.3125 lbs.

Percentage—Page 19

1. $24.00
2. 90.
3. 1.5.
4. 7 159/163%.
5. 150%.
6. 100%.
7. 263.
8. 370.37.
9. $285.71.
10. 16.8 h.p.

Weights and Measures—Page 22

1. 2520 in.
2. 5 yds., 1 ft., 8 in.
3. 3 miles, 1160 ft.
4. 11 miles, 4620 ft.
5. 52 ft. 6 in.
6. 3840 acres.
7. 94 sq. in.
8. 26,666 sq. yds. 6 sq. ft.
9. 11,025 sq. ft.
10. 4,200 sq. rds.
11. 52.92 cu. ft.
12. 221,184 cu. in.
13. 2,376 cu. ft.
14. 250,560 cu. in.
15. 113,280 gr.
16. 32,000 oz.
17. 93 tons, 24,000 oz.
18. 100 cwts.
19. 102,768 oz.
20. $16\frac{2}{3}$ pwt.
21. 104 oz. 80 gr.
22. 6 lbs. 3 oz.
23. 60 5/12 oz.
24. 1,152,000 gr.
25. 10 ser.
26. 25 dr.
27. 7 lbs., 5 7/12 oz.
28. 84 oz.
29. 792 dr.
30. 8 bbls.
31. 21 qts.
32. 2,016 gills.
33. 1 bu.
34. 4096 pints.
35. 5 gal. 20 gills.
36. 144 qts.
37. 100 pecks.
38. 2560 pints.
39. 324,000 sec.
40. 6 deg. 40 min.
41. 290,700 sec.
42. 60 min.

43. 1,296,000 sec.
44. 5,113½ days.
45. 9 yrs.
46. 2,700 min.
47. 648 hrs. 38,800 min.
48. 1,051,920 min.
49. 104.8 m.m.
50. 4.7244″.

Ratio and Proportion—Page 25

1. 533⅓.
2. 2.36 gal.
3. $1,333⅓.
4. $.45.
5. 24 T.
6. 10 days.
7. 16 days.
8. 13⅓ days.
9. 2⅓ days.
10. 902.87 days.
11. 90 men.
12. $319.3.
13. $0.196.
14. 3 3/224 days.
15. 99.5 oz.

Taper Calculations—Page 27

1. 0.052″.
2. 0.150″.
3. 0.050″.
4. 0.130″.
5. 0.250″.
6. 0.03125″ or 1/32″.
7. 0.125″.
8. 0.350″.
9. 0.139″.
10. 0.078″.

Interest—Page 29

1. $70.83.
2. $276.45.
3. $603.75.
4. $571.10.
5. $1,371.60.
6. $232.81.
7. $517.44.
8. $571.20.
9. $3,540.87.
10. $1,238.99.

Pulley and Gear Diameters—Page 31

1. 256 r.p.m.
2. 616 r.p.m.
3. 166⅔ r.p.m.
4. 32 to 1.
5. 20 to 9.
6. 6 in.
7. 1000; 480; 257½; 133⅓; 62½; 30; 16 1/14; 8⅓.
8. 729 to 1.
9. 13.78; 22.96; 38.27; 63.78; 108; 180; 300; 500.
10. 83⅓ and 22.7 f.p.m.

Square Root, Involution and Evolution—Page 35.

1. 496.
2. 364.
3. 222.
4. 157.
5. 21.5139.
6. 1,111.
7. 387,420,489.
8. 6.
9. 823,543.
10. 15,625.

Square Root and Triangulation—Page 36

1. 16.155 ft.
2. 121.037 ft.
3. 224.4 ft.
4. 0.53 in.
5. 180.278 ft.
6. 2.862 in.
7. 8.347 in.
8. 5.887 in.
9. 3.250 in.
10. 3.606 in.
11. 9.540 cu. in.
12. 5.385 ft.
13. 5 ft.
14. 77.518 in.

Cube Root—Page 39

1. 33,076,161.
2. 1,953,125.
3. 161,051.
4. 29.
5. 999.
6. 46.369.
7. 0.75.
8. 80.

Circles—Page 42

1. 0.663 sq. in.
2. $1\frac{1}{8}$ in.
3. 42 sec.
4. 37.699 in.
5. 5.67 pints.
6. 282.1 gals.
7. 5 in.
8. 188.69 sq. in.; 48.695 in.
9. 28.274 sq. in.
10. 0.188 in.
11. 11,781 lbs.
12. 82.467 sq. in.
13. 720 sq. in.
14. 1.1416 sq. in.
15. 1114 sq. in.
16. 0.090 sq. in.

Mensuration and Geometry—Page 49

1. 9 sq. in.
2. 4,800 sq. ft.
3. $48.00.
4. $21.75.
5. 12 sq. in.; 7.688 sq. in.; 15 sq. in.
6. 14.697 sq. in.
7. 20 deg.; 72 deg.; 138 deg.
8. 28 deg.
9. 423.014 gal.
10. 1512.
11. 1.299 cu. in.
12. 28.274 sq. in.; 14.137 cu. in.
13. 395.842 sq. in.; 518.250 cu. in.
14. 18.617 cu. yds.
15. 44.108 lbs.
16. $1.885.
17. 113.097 sq. in.; 43.756 in.
18. 43.982 cu. in.
19. 60 cu. in.
20. 33.6 cu. in.
21. 1.302 in.
22. 2.351 in.
23. 9.184 in.
24. 1.663 in.

Review Exercises—Page 51

1. 1,795.20 gal.
2. 1,584 paces.
3. 7,920 ft.
4. $16.00.
5. 21,656¼ lbs.
6. 385.9 bu.
7. 1.6 h.p.
8. 106⅔ ft.
9. 3.543 in.
10. 4,500 lbs.
11. 5.148 gr.
12. 4,021.45 h.p.
13. 22.88 lbs.
14. 113,588,000 ft. lbs.
15. 4.584 lbs.
16. 1.816 in.

Formulas and Algebraical Expressions—Page 57

1. (a) $9a - 2b + x$. (b) $3a + 4b + 3c$.
2. (a) $2a^2 - 2b - 2c$. (b) $8xy^2z^3$.
3. (a) $a^2 + 10a + 24$. (b) $x^2 - y^2$.
4. (a) $2b$. (b) $4xy$.
5. $6\frac{1}{4}$.
6. 7.
7. -2.
8. 6.333″.
9. 128.57 tons.
10. 13.708 b.h.p.
11. $S = \dfrac{33000 \text{ h.p.}}{U.W.}$; $V = \dfrac{33000 \text{ h.p.}}{S.W.}$; $W = \dfrac{33000 \text{ h.p.}}{S.U.}$.
12. $P = \dfrac{33000 \text{ h.p.}}{L.A.N.}$; $L = \dfrac{33000 \text{ h.p.}}{P.A.N.}$; $A = \dfrac{33000 \text{ h.p.}}{P.L.N.}$;

$$N = \frac{33000 \text{ b.p.}}{P.A.N.}.$$

13. $a = \dfrac{A + \pi c.d.}{\pi b}$; $b = \dfrac{A + \pi c.d.}{\pi a}$;

$$c = \frac{\pi a.b. - A}{\pi d}; \quad d = \frac{\pi a.b. - A}{\pi c}.$$

14. $O = 2U - 2V.X. - 2T.U.Z. + 2M.U.Z. + 2\,\dfrac{N.V.Z.}{P}$.
15. $A = \dfrac{W.P.}{V + S}$; $V = \dfrac{W.P.}{A} - S$; $S = \dfrac{W.P.}{A} - V$; $P = \dfrac{A(V + S)}{W}$.
16. $t = \sqrt{\dfrac{2S}{g}}$; $g = \dfrac{2S}{t^2}$.
17. $V = \sqrt{2g.s. + v^2}$; $v = \sqrt{V^2 - 2g.s.}$; $g = \dfrac{V^2 - v^2}{2S}$.
18. $H = \sqrt{A^2 + B^2}$; $A = \sqrt{H^2 - B^2}$; $B = \sqrt{H^2 - A^2}$.
19. $M = \dfrac{\sqrt{Z.S}}{N^2}$ or $\dfrac{\sqrt{2S}}{N}$; $N = \dfrac{\sqrt{Z.S}}{M^2}$ or $\dfrac{\sqrt{Z.S}}{M}$; $S = \dfrac{M^2.N^2}{Z}$.
20. $H = \dfrac{D(N - M)}{5}$; $N = \dfrac{5H - D.M}{D}$; $M = \dfrac{5H - D.N}{D}$.

Progression—Page 63

1. 4.
2. 13.
3. 50.
4. 8.
5. 2.
6. 729.
7. 4.
8. 2.
9. 29.514.
10. 1.15.

Trigonometry—Page 71

1. 3.8992 in.; 4.6631 in.; 4.0037 in.
2. 11.230 in.; 14.1138 in.; 12.1240 in.
3. 9 deg. 28 min.; 18 deg. 26 min.; 11 deg. 19 min.
4. 7 deg. 17 min.—R.; 2 deg. 43 min.—L.
5. 4 deg. 33 min.
6. 115.47 ft.
7. 1.6643 miles.
8. 0.3492 in.
9. 0.5773 in.
10. 3.4641 in.

Trigonometry (*Continued*)—Page 72

1. 0.6946 in.; 5.4941 in.; 3.1187 in.
2. 8.6933 in.; 20.8321 in.; 24.0488 in.
3. 22 deg. 1 min.; 25 deg. 41 min.; 30 deg.
4. 12 balls.
5. 1.466 in.
6. 52.303 ft.
7. 24 deg.
8. 860.24 ft.
9. 1.342 in.; 2.684 in.
10. 1″ × 3″ × 2.732″.
11. 60 deg. 42 min.
12. 1,464.1 ft.
13. 14.874 miles.
14. 3 deg. 48 min.
15. A = 75 deg.; a = 5.796 in.; b = 1.553 in.
16. B = 76 deg.; a = 0.212 in.; c = 0.878 in.
17. A = 29 deg. 56 min.; B = 60 deg. 4 min.; c = 0.935 in.
18. B = 48 deg.; b = 8.918 in.; a = 8.030 in.
19. 1.9109″ A–B; 2.0169″ A–C; 1.732″ C–D; 2.4397″ C–E.
20. 0.8505″ V; 1.5155″ W; 0.866″ X; 2.375″ Y; 0.8838″ Z.

Feeds and Speeds—Page 76

1. 28.798 r.p.m.
2. 57.268 ft.
3. 6.87 min.
4. 2.78 min.
5. 31.8 r.p.m.
6. 94.248 f.p.m.
7. 53⅓ min.
8. 3,819.7 r.p.m.
9. 382 r.p.m.
10. 349.8 r.p.m.
11. 78.54 f.p.m.
12. 125.664 f.p.m.
13. 5 min. 13 sec.
14. 1 min. 28 sec.
15. 8 hrs. 20 min.
16. 29 hrs. 24 min.
17. 98.2 r.p.m.
18. 108.974 f.p.m.

Cost Calculation—Page 87

1. \$28.50.
2. \$380.01.
3. \$3.12.
4. \$5.77.
5. \$65.20.
6. \$24.45.
7. \$2.40.
8. \$8.48.
9. \$0.51.
10. \$5.91.
11. \$6.76.
12. \$8.66.
13. \$4.96.
14. \$1.60.
15. \$230.90.
16. \$12.51.

Levers—Page 90

1. 52.08 lbs.; 52.08 lbs.
2. 0.352 ft.
3. 400 lbs.
4. 108 lbs.
5. 2 ft.
6. 16.5 in.
7. 207.85 lbs.
8. 140 lbs.
9. $173\frac{1}{3}$ lbs.
10. 4712.4 lbs.

Pulleys—Page 93

1. 1.
2. 181.82 lbs.
3. 75 lbs.
4. 75 lbs.
5. 50 lbs.
6. 480 lbs.
7. 100 lbs.
8. 2,880 lbs.
9. 29.41 lbs.
10. 7.4% or 6.667 lbs.

Screws—Page 95

1. 28,274.4 lbs.
2. 15,079.68 lbs.
3. 9 in.
4. 35,185.92 lbs.
5. 19,792.08 lbs.
6. $\frac{1}{2}$ in.
7. 5.68 in.
8. 4.

Inclined Planes—Page 97

1. 812.27 lbs.
2. 888.89 lbs.
3. 9,178.25 lbs.
4. 618.41 lbs.
5. 184.89 lbs.
6. 678.86 lbs.
7. 326.35 lbs.
8. 5%.
9. 15%.
10. $44\frac{1}{2}$%.

Wedges—Page 98

1. 20 lbs.
2. 20 ins.
3. 4000 lbs.
4. 4.69 lbs.
5. 75 lbs.

Spur Gearing—Page 104

1. 6 P.
2. 0.250".
3. 0.100".
4. 0.100".
5. 4.250".
6. 5".
7. $5\frac{1}{3}$".
8. 4.200".
9. 32.
10. 9.868".
11. 15.708".
12. 18.850".
13. 0.0224".
14. 3.750".
15. 0.262".
16. 4.200".
17. 0.0157".
18. 5".
19. N = 50, C.P. = 0.3142". T = 0.1571", Add. = 0.1000", Ded. = 0.1000", Clear. = 0.0157", Wh.d. = 0.2157", Wg.d. = 0.2000", O.D. = 5.2000".
20. Gear $\begin{cases} \text{N} = 60 \\ \text{P.D.} = 6.000'' \\ \text{O.D.} = 6.200'' \end{cases}$ Pinion $\begin{cases} \text{N} = 40 \\ \text{P.D.} = 4.000'' \\ \text{O.D.} = 4.200'' \end{cases}$
21. Gear $\begin{cases} \text{N} = 52 \\ \text{P.D.} = 6.500'' \\ \text{O.D.} = 6.750'' \end{cases}$ Pinion $\begin{cases} \text{N} = 30 \\ \text{P.D.} = 3.750'' \\ \text{O.D.} = 4.000'' \end{cases}$
22.

	Driver	Driven on J.S.	Driver on J.S.	Driven
P.D....	2"	3"	1.5"	3.5"
O.D....	2.2"	3.2"	1.7"	3.7"

23. No. 5, No. 3.

Bevel Gearing—Page 107

1. 2.309".
2. 57 deg. 31 min.
3. 0.770".
4. 2 deg. 29 min.
5. 2 deg. 29 min.
6. 4.094".
7. 4".
8. 0.2157".
9. 0.174".
10. 45 deg. 10 min.
11. 26 deg. 42 min.
12. 0.240".
13. 3.333".
14. 7.263".
15.

D.D.	Add. and Ded.	Clear.	Wh.D.	Wg.D.	T	T at Small End	Ang. Add.
5"	0.100"	0.0157"	0.2157"	0.200"	0.1571	0.1047	0.050"

Width of Face	P.C.R.	Add. and Ded. Angle	Ded. Ang.	Face Angle	O.D.	Cutting Angle
0.962"	2.8868"	1° 59'	1° 59'	28° 1'	5.100"	58° 2'

16. Gear $\begin{cases} \text{O.D.} = 5.188'' \\ \text{Face Angle} = 27^\circ\ 42' \end{cases}$ Pinion $\begin{cases} \text{O.D.} = 3.286'' \\ \text{Face Angle} = 55^\circ\ 46' \end{cases}$
17. Gear $\begin{cases} \text{P.C. Angle} = 68^\circ\ 12' \\ \text{Face Angle} = 20^\circ\ 1' \\ \text{Cutting Angle} = 66^\circ\ 25' \end{cases}$ Pinion $\begin{cases} \text{P.C. Angle} = 21^\circ\ 48' \\ \text{Face Angle} = 66^\circ\ 25' \\ \text{Cutting Angle} = 20^\circ\ 1' \end{cases}$
18. No. 2 cutter.

Worm Gearing—Page 111

1. 1.906".
2. 3.076".
3. 3.368".
4. 2.038".
5. 6° 28'.
6. 0.885".
7. 0.054".
8. 10 to 1.
9. 0.452.
10.

$$\text{Worm}\begin{cases}\text{Lead} = 0.4000''\\ \text{Add.} = 0.0637''\\ \text{Wh.D.} = 0.1373''\\ \text{Tooth Angle} = 5^\circ\ 18'\\ \text{P.D.} = 1.3727''\\ \text{R.D.} = 1.2254''\\ \text{I.} = 0.062''\end{cases}$$

$$\text{Worm Wheel}\begin{cases}\text{C.P.} = 0.2000''\\ \text{P.D.} = 2.0372''\\ \text{Th.D.} = 2.1645''\\ \text{R. of Throat} = 0.6227''\\ \text{O.D.} = 2.3898''\end{cases}$$

Spiral Gearing—Page 114

1. $D = 3.535''$, $O = 3.735''$, $W = 0.2157''$, Add. $= 0.100''$, $C = 3.535$, $L = 11.107''$.
2. $DA = 7.201''$, $Da = 2.773''$, $NA = 36''$, $Na = 24''$, $OA = 7.401''$, $Oa = 2.973''$, No. of cutter for A = No. 1, No. of cutter for a = No. 3, $C = 4.987''$.
3. $O = 3.431''$.
4. $C = 9.429''$.
5. $D = 4.713''$, $N = 20$, N.C.P. $= 0.5236''$, No. of cutter = No. 2, $C = 4.713''$.
6. $DA = 3.224''$, $Da = 1.274''$, $OA = 3.391''$, $Oa = 1.441''$, C.P.A. $= 0.4221''$, C.P.a. $= 0.3337''$, P.N.A. $= 0.2618''$, P.N.a. $= 0.2618''$. No. of cutter $A = 2$, No. of cutter $a = 5$, $NA = 0.24$, $\alpha A = 51^\circ\ 40'$.
7. $1\frac{1}{4}$ to 1.

Review Exercises—Page 114

1. $18\frac{2}{3}$ ft.
2. 585 ft.
3. \$2.06.
4. 58.81 lbs.
5. 11.018 in.
6. 566 balls—525 & 500.
7. 3.215 in.
8. 9.78 bbls.
9. 502.75 in.
10. $x_1 = 1.526$ in.; $x_2 = 2.181$ in.; $x_3 = 2.269$ in.
11. 24.
12. 242.
13. 1.375.
14. 11.61 lbs.
15. 18.38 lbs.

Dovetails—Page 117

1. 4.100 in.
2. 0.2925 in.
3. 8.358 in.
4. 1.957 in.
5. 2.167 in.

Screw Threads—Page 121

1. V = 0.108″, U.S.S. = 0.081″.
2. U = 0.072″, U.S.S. = 0.054″.
3. 0.620″.
4. 0.892″.
5. 0.468″.
6. 0.031″.
7. 0.267″.
8. 0.0179″, 0.0556″.
9. 0.125″, 0.750″.
10. Triple th'd. 0.125″.
11. D = 1.375″, p = 0.1666″, d = 0.1443″, PD = 1.2307″, RD = 1.0864″.
12. 0.584″.
13. 2.811″.
14. 2.443″.
15. 2° 17′.

Lathe Change Gears—Page 124

1. 65.
2. 69.
3. 30.
4. 5.50″.
5. 120.
6. 60/50.
7. 45.
8. 6 2/5.
9. 25.6.
10. $5\frac{1}{2}$.

Indexing—Page 128

(Note.—Problems in indexing can have more than one answer)
(Below is shown only one answer for each problem)

1. 3 turns 5 holes in 15 hole circle.
2. 1 turn 9 holes in 21 hole circle.
3. 2 holes in 17 hole circle.
4. 8 holes in 17 hole circle.
5. 8 holes in 23 hole circle.
6. +21/23−11/33.
7. +10/21−10/33.
8. +23/29−11/33.
9. +6/2−6/33.
10. +13/39−3/49.
11. Spindle 48, worm 24, 2 idlers.
12. Spindle 40, worm 56, 2 idlers.
13. Spindle 40, worm 72, 1 idler.
14. Spindle 48, 1st stud 24, 2d stud 40, worm 72, 1 idler.
15. Spindle 64, 1st stud 24, 2d stud 24, worm 72, no idlers.
16. 1 turn 8 holes in 18 hole circle.
17. 1 turn 7 holes in 27 hole circle.
18. 16 holes in 27 hole circle.
19. 2 turns 7 holes in 18 hole circle.
20. 23 holes in 27 hole circle.

Spiral Milling—Page 130

(One answer for each problem)

1. Driven 72×64, driver 24×40.
2. Driven 72×64, driver 48×24.
3. Driven 24×40, driver 64×100.
4. 10 in.
5. 12½ in.
6. Driven 64×40, driver 32×100, 38 deg. 9 min.
7. 15 deg. 31 min.
8. Driven 72×64, driver 24×32.
9. Driven 56×64, driver 28×40.
10. Driven 48×24, driver 24×32.

Friction—Page 132

1. 0.286.
2. 0.188.
3. 0.179.
4. 1 deg. 12 min.
5. 11 deg. 19 min.
6. 25 deg. 38 min.
7. 11 deg. 19 min.
8. 75 lbs.
9. 87½ lbs.
10. 33 lbs.

Electricity—Page 134

1. (*a*) 13.073 ohms.
   (*b*) 50,484.7 ft.
2. 0.638 ohms.
3. 2.4 ohms.
4. 1.1 ohms.
5. 8.8 ohms.
6. 137.5 amps.
7. 4.4 amps.
8. 11 kilowatts.
9. 2,000 volts.
10. 1.5 volts.
11. 70 volts.
12. 0.02286 amps.
13. 5½ cents.
14. $13.14.

Horsepower Calculation—Page 138

1. 510,625 ft. lbs.
2. 175 lbs.
3. 0.141 h.p.
4. 1,042.55 h.p.
5. 8.06 h.p.
6. 0.60 h.p.
7. 5.641 in.
8. 12.8 h.p.
9. 14.77 h.p.
10. 6.4 h.p. (Formula No. 2).
    5.38 h.p. (Formula No. 3).

Strength and Proportion of Gear Teeth—Page 139

1. 47.85 h.p.
2. 0.623 h.p.
3. 3 P or 1 in C.P. cutter.
4. 4.299 in.
5. 2 P cutter.
6. 2.006 in.
7. 0.897 h.p.
8. 1.25 h.p.
9. 9.958 in.
10. 1½ P cutter.

Resolution of Forces—Page 141

1. 49,500,000 ft. lbs. per min.
2. 600 lbs., no direction ($R = 0$).
3. 24 min., B's direction or west Shore.
4. Tension 2.446 tons, compression 9.534 tons.
5. 70.71 lbs., 135 deg. with A in N.W. direction.
6. 22.36 lbs., 26 deg. 34 min. with vertical, on opposite side of the other component.
7. 707.11 lbs.
8. 223.61 lbs., 116 deg. 34 min. with 1000 lbs. force in N.W. direction.
9. 5,315.07 lbs., 131 deg. 11 min. with smaller force in N.W. direction.
10. 50 lbs.

Falling Bodies—Page 144

1. 160.8 ft.
2. 3,618 ft., 14,472 ft.
3. 7.89 sec.
4. 80.2 ft. per sec.
5. 38.87 ft., 155.47 ft.
6. 113.42 ft. per sec., 3.53 sec.
7. 3.11 sec., 155.47 ft.
8. 1,608 ft., 321.6 ft.
9. 476.92 ft., 159.71 ft.
10. 188.94 ft. per sec.

Centrifugal Force—Page 145

1. 3,068.89 lbs.
2. 1,990.06 lbs.
3. 127.87 lbs.
4. 777.13 lbs.
5. 244.39 lbs.
6. 721.8 ft. per min.
7. 24.25 ft.
8. 25,082.92 lbs.
9. 218.57 lbs.
10. 170.50 lbs.

Horsepower of Belting—Page 147

1. 24 h.p.
2. 5.566 in.
3. 35 h.p., 60 h.p.
4. 6.111 in.
5. 5.893 in.
6. 49.09 h.p.
7. 22.5 h.p.
8. 1.6 in.
9. 5.5 in.
10. 2.357 in.

Length of Belting—Page 148

1. 59.82 ft.
2. 34.16 ft.
3. 40.03 ft.
4. 45.35 ft.
5. 55.22 ft.
6. 36.9 ft.
7. 33.76 ft.
8. 35.18 ft.
9. 36.16 ft.
10. 74.81 ft.

Rope Drives—Page 150

1. 15 h.p.
2. 60 h.p.
3. 234.38 lbs.
4. 0.675 lbs., 2.7 lbs.
5. 339.29 h.p.
6. 5 ropes.
7. 13 ropes.
8. 487.5 lbs.
9. 5 ropes.
10. 3.288 in.

Cable or Wire Rope Drives—Page 151

1. 300 h.p.
2. 187.5 h.p.
3. 1.58 lbs., 3.57 lbs.
4. 622.125 lbs.
5. 1 cable.
6. 216.46 h.p.
7. 1 cable.
8. 2 +cables.
9. 1.558 in.
10. 26.77 lbs.

Chain Transmission—Page 153

1. 3.236″.
2. O.D. = 6.084″, B.D. = 5.434″.
3. A = 1.050″, B = 0.700″, b = 0.569″.
4. P.D. = 14.349″, O.D. = 14.837″, B.D. = 13.861″.
5. 7.201″.

Shaft Design—Page 156

1. 1.08 deg. (1 deg. 4.8 min.).
2. 1.138 in.
3. 70 h.p.
4. 3.868 in.
5. 5.21 ft., 8.24 ft.
6. 12.66 h.p.
7. 7.65 in.
8. 0.0284 deg. (1.7 min.).
9. 10.80 ft.
10. 12.5 in.

Bearing Design—Page 158

1. 571 lbs. per sq. in.
2. 6.349 in.
3. 20.089 in., 13.393 in.
4. 3.438 in.
5. 10.02 in.
6. 5.006 in.
7. 157.5 sq. in.
8. 7.209 in.
9. 3.250 in., 2.167 in.
10. 7.5 in., 5.0 in.

Ball Bearing Design—Page 160

1. 130 lbs.
2. 13 balls, 26 balls.
3. 0.130 in.
4. 0.306 in.
5. 0.223 in., 0.158 in.
6. 12 balls.
7. 1.00 in.
8. 15balls.

Center of Gravity, Moment of Inertia, etc.—Page 165

1. $x = 2.167''$, $y = 1.167''$.
2. $x = 1.926''$, $y = 1.148''$.
3. $x = 4.937''$, $y = 6.214''$.
4. 10.750.
5. 4.302.

Strength of Materials—Page 186

1. 2.236 in.
2. 0.0163 in.
3. 55,200 lbs.
4. 0.249 in.
5. 3.799 in.
6. 36.462 lbs.
7. 2.48 in.
8. 0.020 in.
9. 2.82 in.
10. 30,000,000.

Springs—Page 191

1. 3.072 in.
2. 490.625 lbs.
3. 3.297 in.
4. 0.406 in.
5. $\frac{3}{8}$ in.
6. 0.694 in.
7. O.D. = 2 15/32 in., h = 4.853 in.
8. 40.502 in.
9. W = 6,125 lbs., δ = 1.6 in.
10. 892.43 lbs.

Pipes and Cylinders—Page 193

1. 0.36 in.
2. 125 lbs.
3. 0.6 in.
4. 0.3 in.
5. 1,318.75 lbs.

Riveted Joints—Page 196

1. $d = \frac{3}{4}''$ (or $\frac{3}{16}''$), $p = 1\frac{3}{16}''$ (for $\frac{3}{4}$ rivet).
2. $\frac{11}{16}''$.
3. $1''$, $3\frac{1}{4}''$.
4. 0.692.
5. $d = \frac{13}{16}''$ or $\frac{7}{8}''$, $p = 1''$ (for $\frac{13}{16}''$ rivet).
6. 47,250 lbs.

Logarithms—Page 199

1. $\bar{2}.6902$; $\bar{1}.6902$.
2. 3.5128; $.\bar{1}5128$.
3. 1,920.
4. 93.9.
5. 0.003254.
6. 4.350.
7. 0.03408.
8. 433.4.
9. 2,159.
10. 3.48.
11. 2,073.
12. 75.2.
13. 7.663.
14. 0.3079.
15. 57.07, 599.4.
16. 3.785, 4.045.
17. 68.11, 618,000.
18. 0.007673.
19. 400.9.
20. 0.0005986.

Heat—Page 204

1. (*a*) 73 2/5° F.
   (*b*) 44 3/5° F.
   (*c*) 5° F.
2. (*a*) 17 2/9° C.
   (*b*) 9 4/9° C.
   (*c*) 20° C.
3. (*a*) 672° F. (abs.).
   (*b*) 420° F. (abs.).
   (*c*) 285° C. (abs.).
4. 18.63 lbs.
5. 202 deg. F.
6. 7.6 lbs.
7. 10,340 B.t.u.
8. 1.34 gallons.
9. 14.67 h.p.
10. 172.5 lbs.
11. 78.8 lbs.

Metal Cutting—Page 209

1. 3.5 h.p.
2. 1.17 h.p.
3. 1.14 h.p.
4. 0.28 h.p.
5. 1,325.36 lbs.
6. 47 ft. per min.
7. 30.4 ft. per min.
8. $\frac{1}{16}$ in. per rev.

Force, Work, Energy and Momentum—Page 214

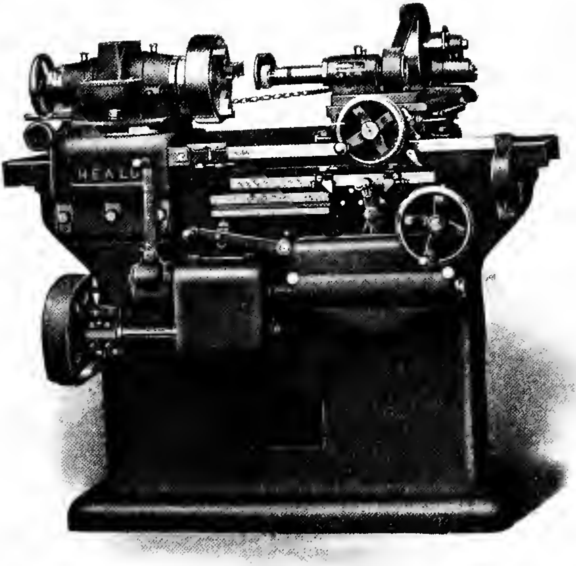
1. 3,498,134 ft. lbs.
2. 3,498,134 ft. lbs.
3. 309.4 ft. lbs.
4. 276,631.9 lbs.
5. 165 lbs.
6. 35.28 kilowatts.
7. 127.3 h.p.
8. 30.8 h.p.
9. 42.9 lbs. per ton.
10. $106\frac{2}{3}$ h.p.
11. 12,000 lbs.
12. 8 h.p.
13. 850 lbs.
14. 48,500 lbs.
15. 27.78 ft. per sec.
16. 850 lbs., 14.1 h.p.

Pendulum—Page 230

1. 39.1 in.
2. 32.1909 ft. per sec. per sec.
3. 0.96 sec.
4. 18.04.
5. 1.226 sec.
6. 1.061 sec.
7. 0.1026 in.
8. 1.023 sec.
9. 2.048 rev. per min.
10. $t_3 = 0.75$ sec., $r = 0.361$ ft., $h = 0.458$ ft., $\alpha = 38\frac{1}{4}°$.

Review Exercises—Page 236

1. 3,819.7 rev. per min.
2. 6,021.4 ft. per min.
3. 125.
4. 125.
5. 0.000355 in.
6. 141, 372 lbs.
7. 1,380.6 lbs.
8. 31,406.8 lbs.
9. 3.305 in.
10. 25.



INTERNAL GRINDING MACHINE